













# NAVAL POSTGRADUATE SCHOOL

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## THESIS

Forecasting U.S. Army Major Command Readiness Based  
on Enlisted Personnel Strength

By

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December, 1983

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Module ages the MACOM's using the theory of Markov Chains. Finally, the Readiness Indicator Module computes and assigns a readiness rating to the MACOM's based on the personnel criteria specified in AR 220-1. The results obtained from this methodology can aid DCSPERS, MILPERCEN, and other decision makers in the formulation of future manpower policies concerning recruitment, promotion, expiration term of service (ETS) and permanent change of station (PCS).



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Forecasting U.S. Army Major Command  
Readiness Based on Enlisted Personnel Strength

by

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Submitted in partial fulfillment of the  
requirements of the degree of

MASTER OF SCIENCE IN MANAGEMENT



## ABSTRACT

A model using linear programming optimization and Markov Chain forecasting techniques is presented to forecast future Major Command (MACOM) readiness based on the personnel criteria of Army Regulation (AR) 220-1. The model is composed of four modules. First, the Recruitment Module forecasts accessions based on total numbers recruited and historical attrition rates. Second, the Distribution Module optimally assigns all new accessions and permanent change of station moves to the MACOM's. Next, the Forecasting Module ages the MACOM's using the theory of Markov Chains. Finally, the Readiness Indicator Module computes and assigns a readiness rating to the MACOM's based on the personnel criteria specified in AR 220-1. The results obtained from this methodology can aid DCSPERS, MILPERCEN, and other decision makers in the formulation of future manpower policies concerning recruitment, promotion, expiration term of service (ETS) and permanent change of station (PCS).





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## I. INTRODUCTION

The United States Army defines force readiness

"as measured by its ability to man, equip, and train its forces and to mobilize, deploy, and sustain them in accomplishing assigned missions." [Ref. 1: p. 1-1]

An objective assessment of current force readiness and the ability to forecast and assess future force readiness is of paramount importance to the Department of Defense. Domestic and foreign policy and strategic and tactical plans for the defense of the United States are based on projections of future force readiness.

### A. THE UNIT STATUS REPORT

The U.S. Army currently uses the Unit Status Report (USR) to assess force readiness. The objectives of the USR are twofold. First, the USR is to provide a current status of U.S. Army units to National Command Authorities (NCA), the Joint Chiefs of Staff (JCS), Headquarters, Department of the Army (HQDA), and to all chain of command levels. Secondly, the USR is to provide four indicators to the Department of the Army (DA). The first indicator identifies factors which degrade unit status. Next, it is to assist the DA and intermediate commands in allocating resources. Thirdly, it is to assist in identifying differences between current personnel and equipment assets in units and full





wartime requirements. Finally, the USR provides indicators which assist in determining Army-wide conditions and trends. [Ref. 1: p. 1-1]

The USR assigns a unit readiness rating to all reporting units based on the three broad areas of personnel, logistics and maintenance, and training. A brief explanation of the USR in the area of personnel will follow. The reader may reference Army Regulation (AR) 220-1 for more details in the areas of logistics and maintenance and training.

Personnel readiness as defined by the USR is composed of three objectively rated components. The first is percent personnel fill (PPF) which is the number of personnel on hand divided by the required number of personnel. The second component is the percent senior grade fill (SGF) which is the ratio of the total number of soldiers in grade E5 through the highest officer rank authorized for the unit divided by the required number of such personnel. The last component is percent MOS fill. MOS stands for a military occupational specialty that is awarded to a soldier upon completion of advanced individual training (AIT). The MOS is specified by a code that refers to a specific type of job and job skill. A more detailed explanation of MOS and its code will be given in Chapter II. It is possible for a soldier to be assigned to a billet and not be MOS qualified for that position. The number of personnel qualified for the billets to which they are assigned, divided by the



required number of personnel in the unit is the percent MOS fill, the third component of the personnel readiness rating.

The Army rates a given units' personnel status by assigning a "C-rating" to the unit. A rating of C1 is the highest. Ratings of C2 and C3 indicate lesser capability. A rating of C4 means that as a whole, the unit is incapable of performing its total mission due to personnel shortages. AR 220-1 defines the C-ratings in the following manner. C1 implies that a unit is combat ready with no deficiencies. A rating of C2 means that the unit is combat ready, although with minor deficiencies. A C3 rating is still classified as combat ready but with major deficiencies. Finally, a C4 means that the unit is not combat ready. [Ref. 1: p. 1-2]

#### B. REPORTING PROCEDURES WITH THE USR

Each battalion and separate company sized unit in the Army completes and submits the USR on a monthly basis. Each submitting unit computes the previously defined ratios PPF, SGF, and percent MOS fill. By consulting with AR 220-1, the unit assigns a C-rating to each of the personnel areas. The overall rating for personnel readiness becomes the lowest rating of its three components: PPF, SGF, and percent MOS fill. If the personnel rating is lower than C1, the unit must report the reason for the lower rating. For example, if a particular unit is assigned the ratings of C1 for PPF and percent MOS fill and C2 for SGF, this unit would then



report its overall personnel readiness rating as "C2 for SGF." All higher echelon units that received this report would immediately know that this unit's major shortcoming is a shortage of senior grade personnel.

Brigade sized units accept and analyze the USR's from the battalions and separate companies. Each USR is then collated and forwarded to the next higher command along with comments. This process continues successively until the reports reach the HQDA. It is important to note that only battalions and separate companies are assigned a personnel readiness rating. However, policy and decision makers at the Department of the Army can easily impute a readiness rating for all intermediate commands by simply observing the ratings of all rated units subordinate to the intermediate command in question.

To give meaningful recommendations to the Department of Defense for policy and strategic planning, the Department of the Army must be able to accurately forecast future personnel readiness. The proper allocation of future personnel resources depends heavily on accurate forecasts of future personnel readiness. [Ref. 1: p. 1-1] This thesis will attempt to develop a mathematical model that will forecast future unit readiness based on the personnel criterion only. The method will be demonstrated using data concerning personnel of a single MOS only. No attempt will be made to model the two other previously mentioned areas of





unit readiness or to forecast unit readiness based on the criteria for logistics and maintenance and training.

### C. THESIS OUTLINE

Chapter II of this thesis provides the reader with an overview of the problem description and model formulation, in general, starting with a section devoted to definition of terms needed for a full understanding of the problem. Chapter III will specify the data required and give the reader an appreciation for the complexity of the data base needed to support this model. Chapter IV will be devoted to methodology. It will describe the mathematical details of the model to be developed. In Chapter V, a discussion of this model's results will be presented along with comments. Finally, Chapter VI will be reserved for conclusions, and the potential of this model's methodology as a decision making tool will be discussed.



## II. PROBLEM DESCRIPTION AND MODEL FORMULATION

### A. EXPLANATION OF TERMS

This thesis will frequently make reference to MOS's, skill level's, MACOM's, and the TTHS account. A brief explanation of these terms is necessary for a better understanding of the problem description.

#### 1. The Military Occupational Specialty (MOS)

Each soldier is awarded a military occupational specialty (MOS) upon completion of basic individual training (BIT) and advanced individual training (AIT). Each MOS refers to a specific type of job and job skill. All MOS's are denoted by a three character code. The first two characters are digits that refer to the career management field (CMF). Examples are CMF 11 and CMF 12 referring to the fields of infantry and armor, respectively. The third character in the MOS is a letter that further subdivides and classifies the CMF. 11B and 11C respectively denote light infantry and indirect fire (mortar) infantry.

If it is important to classify a given soldier by both MOS and skill level, a five character code is used to specify the MOS and skill level combination. The last two digits of the code then denote the soldier's skill level. A skill level one soldier is signified by 10. Likewise, soldiers in skill levels two through five are assigned the



designators 20 through 50. A skill level is analogous to a pay grade. Pay grades E1 through E4 make up skill level one. Pay grades E5 through E7 correspond to skill levels two through four respectively. Finally skill level five is composed of pay grades E8 and E9. To continue the previous example, 11B20 and 11C30 denote a skill level two (E5) light infantryman and a skill level three (E6) indirect fire infantryman.

## 2. Skill Level

The most commonly known rank structure is the pay grade. Currently, the authorized manning levels of all active units are based on an E3 being the lowest grade in the unit. The Army allows however, any skill level one soldier of requisite MOS to fill an E3 billet and any skill level one soldiers in the pay grades of E2, E3, or E4 are authorized to fill an E4 billet. Thus, USR's submitted by battalions and separate companies contain skill level information rather than rank or pay grade information. For this reason, grade structure in this thesis will be represented by skill level and not the more commonly recognized pay grade.

## 3. The MACOM

The term MACOM is an acronym for major command. There are currently more than thirty MACOM's in the Army. However, the majority of Army personnel are associated with only five MACOM's: the U.S. Army Forces Command (FORSCOM),



the U.S. Army Training and Doctrine Command (TRADOC), the U.S. Army Europe and the Seventh Army (USAEUR), the Eighth U.S. Army (EUSA), and the U.S. Army Western Command (WESTCOM). A sixth MACOM to appear in this thesis is a fictitious MACOM composed of the entire U.S. Army minus the above mentioned five MACOM's. In all future references this MACOM is called OTHER.

#### 4. The TTHS Account

The TTHS account is in essence a personnel overhead account. TTHS is a standard Army abbreviation for trainees, transients, holdees, and students. All soldiers attending OSUT, BIT and AIT, or any other school awarding a skill qualification are classified as trainees. All soldiers, during the conduct of travel from one duty station to another, are considered to be transients. Holdees are prisoners and all soldiers projected for long term hospitalizations. Finally, a soldier is classified as a student if he is a Cadet at the United States Military Academy or if he is attending any school not awarding a military skill qualification.

The TTHS account is considered in the total end strength of the U.S. Army but is not considered in the individual MACOM end strengths. A more detailed explanation of the significance of the TTHS account will be provided in Chapter IV.





## B. OVERVIEW

As the field operating agency for the Deputy Chief of Staff for Personnel (DCSPERS), one of the many missions assigned to the U.S. Army Military Personnel Center (MILPERCEN) is to allocate and assign all of the soldiers in the Army to units. To accomplish this mission, MILPERCEN distributes all skill level one soldiers who are recent graduates of AIT and all soldiers conducting a permanent change of station (PCS) move to MACOM's within the Army. Currently, MILPERCEN distributes soldiers by skill level and MOS to MACOM level only. The individual MACOM's then are required to distribute newly assigned personnel to their subordinate commands. For this reason, distribution data on soldiers below the MACOM level is unavailable from MILPERCEN.

Five more terms need to be defined at this point to more clearly understand the terminology of the model to be developed.

### 1. Permanent Change of Station (PCS)

When a soldier has completed his tour of duty at his current unit, he may be reassigned by MILPERCEN to another unit. This reassignment is called a PCS move. In this model a PCS move is classified as such only if the gaining and losing installations are in different MACOM's.



## 2. Expiration Term of Service (ETS)

When a soldier has completed the length of service to which he is contractually bound and he decides not to re-enlist, then he is said to ETS. An ETS occurs when a soldier separates from the Army with less than twenty years service.

## 3. Retirement

A retirement occurs when a soldier is eligible for ETS, has twenty or more years of service, and he elects not to re-enlist.

## 4. Attrition

The term attrition is used to refer to any soldier who separates from the service. In this model an attrition is defined to be a soldier who separates from the Army because of an ETS move, a retirement, or a failure to graduate from BIT and AIT or One Station Unit Training (OSUT).

## 5. Recruitment

In this model, recruitment is the process of supplying soldiers to MACOM's. Thus, recruits are soldiers newly assigned to any MACOM regardless of skill level. Recruits, therefore, include soldiers who have recently graduated from AIT or OSUT or soldiers who have recently completed a PCS move.

At last count, the U.S. Army was composed of 370 enlisted MOS's. The probabilities of promotion, PCS, ETS,



and retirement vary from one MOS to another. The problem of gathering the required data for all 370 MOS's would have been beyond the scope of this modeling effort. Since the largest enlisted MOS in the Army is light infantry or 11B, it was reasoned that data would be abundant and easily obtainable for that MOS. Therefore, this thesis attempts to develop a methodology to forecast future unit readiness based on the personnel criterion for the enlisted MOS 11B only. It is thought that if the methodology works for the MOS 11B, then it will be found to be appropriate for all MOS's as well.

To better understand the model to be developed, it is helpful to follow the career of a typical 11B enlisted soldier. Figure 1 is a schematic of his career. The typical soldier would start his Army career by enlistment in the Army and attendance of BIT and AIT or OSUT. If the soldier fails at this point in his career he attrits from the Army. Otherwise, he will become part of the Army's manpower pool to be distributed by MILPERCEN to one of the six previously mentioned MACOM's. When his tour is complete at his assigned MACOM, the soldier will either separate through ETS and become part of the Army's attrition or he will rejoin the manpower pool in order to conduct a PCS move to another MACOM. This process may continue many times until the soldier separates from the Army, either through ETS, retirement, or death.



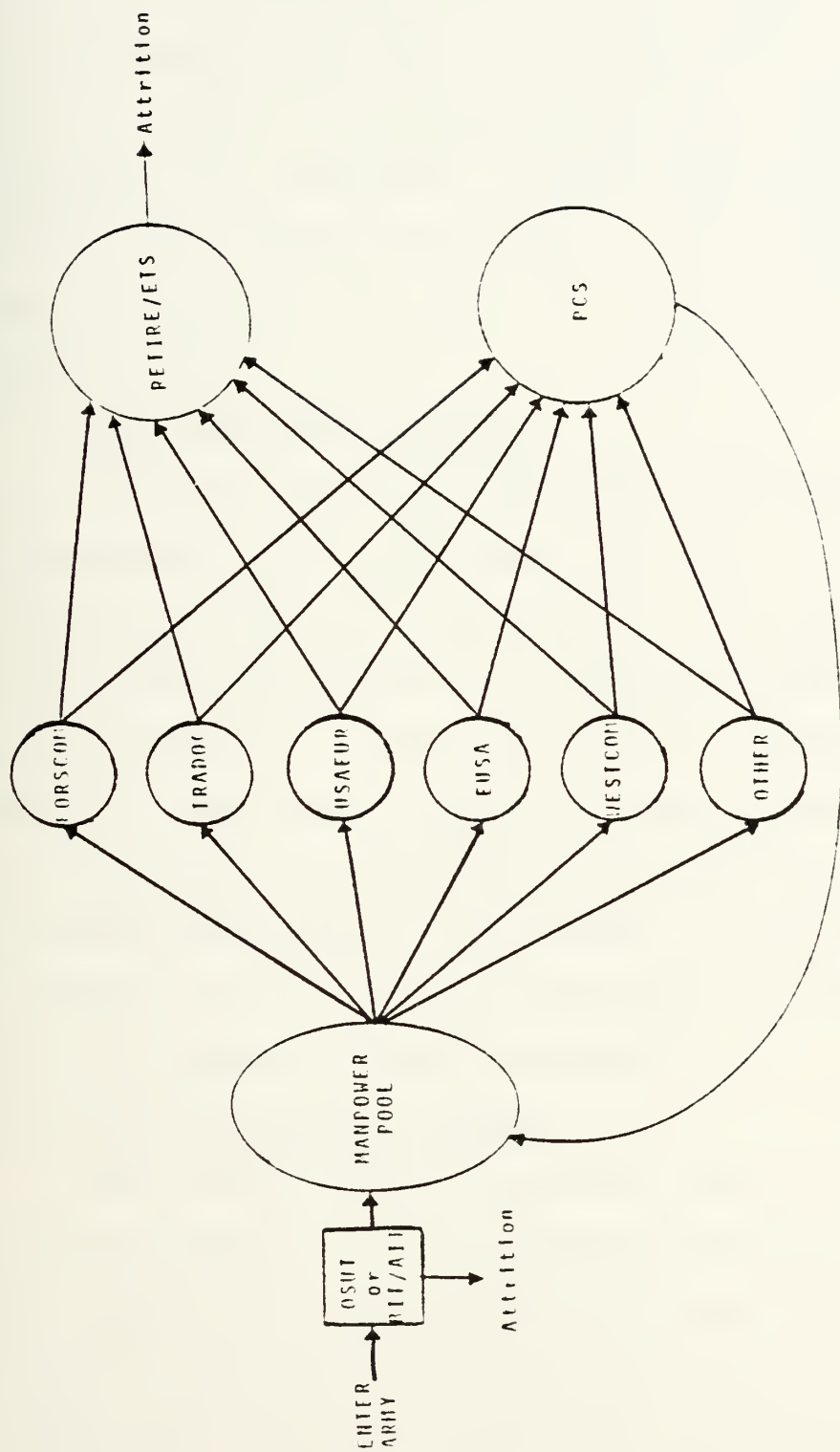


Figure 1. Modeling Method One





## C. THE MODEL

To accomplish the goal of developing a methodology to predict a given MACOM's force readiness based on personnel fill criterion, the model was divided into four modules. The four modules to be discussed briefly in this section are the modules for recruitment, distribution, forecasting, and the readiness indicator module. A brief description of these modules will be followed by a discussion of the assumptions made.

### 1. The Recruitment Module

When an individual joins the Army he is considered an accession to the recruitment module. If he fails to graduate from OSUT or BIT and AIT, he is considered to be attrited from this module. A soldier may graduate from OSUT or BIT and AIT as an E1, E2, or E3. Regardless of pay grade, all soldiers in the recruitment module are skill level one. This fact makes a single attrition factor for this module appropriate. The number of civilians entering this module is multiplied by the course completion rate to obtain the number of soldier graduates.

### 2. The Distribution Module

The output of the recruitment module becomes input to the distribution module. The distribution module is a linear program with the objective of maximizing the minimum level of either PPF or SGF for each MACOM, subject to the personnel fill criteria established by the Department of the



Army. If feasible, the output of the distribution module will be the optimal distribution of personnel to insure that each MACOM in question is assigned at least the minimum amount of personnel as required by current Army policy.

In essence, the distribution module finds the MACOM with the lowest percentage fill in either PPF or SGF and it allocates soldiers in the personnel pool to that MACOM with the appropriate MOS and skill level until there is a tie for the lowest percentage fill. It then distributes soldiers to both of these MACOM's in equal proportions, as set by the model's constraints, until there is a three way tie. This process is repeated until all soldiers available in the pool have been distributed. The output of the distribution module becomes input to the forecasting module.

It is important to point out that the individually distinct or discrete nature of the demand for soldiers (i.e., the number of soldiers must be an integer) by the various MACOM's will not be considered. The assumption will be made that the pool of soldiers to be distributed is a continuous quantity. Thus a linear program to optimally distribute soldiers is assumed to be appropriate.

### 3. The Forecasting Module

The forecasting module is by far the most complex in this model in the sense that the data base required to support it is very large. This module predicts future MACOM



personnel end strengths by skill level using the theory of Markov Chains. [Ref. 2: p. 85]

To understand more clearly the explanation, development, and concept of this module, it is necessary to define several key terms used throughout this module. All time periods in this model are fiscal years.

The system of this module is defined to be the entire U.S. Army enlisted force of MOS 11B (light infantry). The system is composed of six subsystems, each of which corresponds to the previously defined MACOM's. Each of the MACOM's are composed of five skill levels. Thus there are thirty categories in which a soldier of MOS 11B may serve. These categories are defined to be the states of the system.

The basic components of this module are the stocks and flows. At any point in time the soldiers in a MACOM can be classified into skill level categories or states. The number of soldiers in these categories are called this MACOM's stocks at that point in time. When these stocks are placed contiguously in ascending skill level order, a row vector of MACOM stocks is formed. When the row vectors of stocks for all six MACOM's are placed contiguously in ascending MACOM order, the stock vector is formed and is denoted by  $\underline{n}(t)$  where the argument  $t$  represents a point in time, such as the beginning of a fiscal year. The ordering of the MACOM's is important only to the extent that it should be consistent throughout and is given in Chapter IV.



The stock vector provides a snapshot view of the MACOM at time  $t$ . [Ref. 2: p. 3 ]

Flows are movements of soldiers during a time period such as a fiscal year. Flow data captures the dynamic transactions that occur in a MACOM. It is important to observe that flows relate to an interval of time and not a point in time as in the case of stocks. [Ref. 2: p. 3]

It is assumed in this model that there are only five possible transitions that a soldier may undertake during one fiscal year. The soldier may remain in his current skill level and MACOM; if so he is called a "stayer". A soldier may be promoted and remain in his current MACOM in which case he is called a promotee. Thirdly, a soldier may remain in his current skill level and carry out a PCS move. If this occurs, he is classified as a "PCS". Next, a soldier may conduct a PCS move and get promoted to the next higher skill level. This type of transition will be called a PCS and promotion. Finally, a soldier may retire, die, or conduct an ETS move in which case he is classified as an attrition. The first two types of transitions are called intra-MACOM transitions. The PCS and the PCS and promotion moves are called inter-MACOM transitions. Since demotions are infrequent, it is assumed here that they do not occur.

When the flows in a given fiscal year are considered as proportions of stocks present at the beginning of the fiscal year, transition rates are formed. Thus, intra-MACOM





transitions are depicted by staying rates and promotion rates. Likewise, inter-MACOM transitions are depicted by PCS rates and rates of PCS and promotion. Also, all attritions from the Army are transformed this way into attrition rates.

For this model, the forecasting module is assumed to be a Markov Chain. In a Markov Chain, the probability of being in any state depends only on its previous state and not on its prior history. The rates described above are considered as estimates of transition probabilities (to be described in more detail) which govern transitions among states during one time period. It is usually assumed that all transition probabilities are constant over several periods. The procedure for estimating these transition probabilities will be given in more detail in Chapter IV. The validity of the assumption of constancy over several time periods will be discussed in Chapter V. [Ref. 2: p.106]

Finally, it is assumed that for any one individual soldier only one of the above transitions may occur during a fiscal year. This assumption is reasonable as it is virtually impossible for a soldier to transition twice in a fiscal year. For example, it is assumed that a soldier cannot be promoted from skill level one to skill level three in a fiscal year. Since most NCO promotions occur only if the soldier re-enlists or extends his enlistment, the number of soldiers who are promoted to skill level two and conduct



an ETS move in the same fiscal year are very few. Thus, a promotion and an attrition by the same soldier in the same fiscal year are assumed not to occur together. Also it is assumed that a soldier may not conduct two PCS or ETS moves in a fiscal year.

The Markov Chain predicts future stocks in accordance with the basic equation

$$\underline{n}(t) = \underline{n}(t-1) \underline{P} + \underline{r}(t). \quad (1)$$

The stock vector  $\underline{n}(t-1)$  contains, as its components, the number of skill level one through skill level five 11B soldiers in the first MACOM followed by similar quantities for the second through the sixth MACOM's. At the beginning of a fiscal year the stock vector  $\underline{n}(t)$  is dimensionally and structurally the same as the current stock vector,  $\underline{n}(t-1)$ , however,  $\underline{n}(t)$  consists of the projected stocks one time period later. [Ref. 2: p. 90]

Recruits are classified by the same categories as stocks. Therefore, when the number of recruits into a MACOM are placed contiguously in ascending skill level order, a row vector of MACOM recruits is formed. When the row vectors of recruits for all six MACOM's are placed contiguously by MACOM in the same order as the previously described stock vector, the recruitment vector is formed and is denoted by  $\underline{r}(t)$ . The argument  $t$ , again refers to the beginning of the fiscal year in question. It is important



to recall here, that the output of the distribution module when placed in this vector form, is the recruitment vector.

The transition matrix,  $\underline{P}$ , formed of the transition probabilities is a square matrix where each row and column represents one of the categories or states described earlier. Each row of the transition matrix may be viewed as a probability vector that represents the set of all probabilities of transitioning during a fiscal year from the state represented by that given row to another state represented by the column in which the probability is located. [Ref. 3: p. 348]

The transition matrix used in this thesis contains thirty-six sub-matrices. The six sub-matrices located along the main diagonal correspond to the six MACOM's and as such each one contains the intra-MACOM transition probabilities for one of the six MACOM's. The other thirty sub-matrices represent transitions between two different MACOM's, i.e., inter-MACOM transitions. Figure 2 is a schematic of the overall transition matrix. The main diagonal sub-matrices and two typical off-diagonal sub-matrices are shown. The other off-diagonal sub-matrices are suppressed here for the sake of clarity.

The rows and columns of each sub-matrix correspond to the five skill levels in a MACOM. Thus each of the thirty-six sub-matrices is of dimension  $5 \times 5$ . Each row of the sub-matrix is made up of elements denoted by  $p_{ij}$  which



	FORSCOM	TRADOC	USAEUR	EUSA	WESTCOM	OTHER
FORSCOM	FORSCOM SUB-MATRIX			FORSCOM TO EUSA TRANSITIONS		
TRADOC		TRADOC SUB-MATRIX				
USAEUR			USAEUR SUB-MATRIX			
EUSA				EUSA SUB-MATRIX		
WESTCOM		WESTCOM TO TRADOC TRANSITIONS			WESTCOM SUB-MATRIX	
OTHER						OTHER SUB-MATRIX

Figure 2. The Transition Matrix (Schematic)





represent the probability of a soldier transitioning from skill level  $i$  to skill level  $j$  in a fiscal year. Therefore at most two such elements ( $p_{ii}$  and  $p_{ii+1}$ ) are positive while the rest are zero. Techniques for estimating the transition probabilities,  $p_{ij}$ , for the entire transition matrix,  $\underline{P}$ , will be demonstrated in Chapter IV. Figure 3 is an example of a typical sub-matrix.

To demonstrate briefly the forecasting module, consider the current time to be  $t-1 = 0$ . To forecast the future stocks for  $t = 3$ , one would input the current stock vector for  $\underline{n}(t-1)$  and compute  $\underline{n}(1)$  from the basic Markov Chain equation previously stated. The recruitment and distribution modules would be used again until a new recruitment vector for time  $t = 2$ ,  $\underline{r}(2)$ , is obtained. The stock vector  $\underline{n}(1)$ , previously computed, would then become the current stock vector and  $\underline{n}(2)$  would be calculated from the same basic equation. This procedure is then repeated to forecast for time  $t = 3$ .

Two methods of modeling using Markov Chains and optimization to forecast future personnel end strengths and distribute them to MACOM's are possible. The first, which is referred to as modeling method one, is to distribute optimally first term recruits out of AIT and all PCS transfers through the distribution module and then forecast next years' stocks using a very sparse transition matrix where all the off-diagonal sub-matrices in Figure 2 would



	SL1	SL2	SL3	SL4	SL5
SL1	$p_{11}$	$p_{12}$	$\emptyset$	$\emptyset$	$\emptyset$
SL2	$\emptyset$	$p_{22}$	$p_{23}$	$\emptyset$	$\emptyset$
SL3	$\emptyset$	$\emptyset$	$p_{33}$	$p_{34}$	$\emptyset$
SL4	$\emptyset$	$\emptyset$	$\emptyset$	$p_{44}$	$p_{45}$
SL5	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$p_{55}$

If Main Diagonal

$P_{ii}$  = Probability of Remaining a Skill Level  $i$  in Current MACOM During a Fiscal Year.

$P_{ii+1}$  = Probability of Promotion From Skill Level  $i$  to Skill Level  $i+1$  and Remaining in Current MACOM During a Fiscal Year.

If Off Diagonal

$P_{ii}$  = Probability of Skill Level  $i$  PCS Move From Current State (Skill Level & MACOM) to Another State (Same Skill Level & Different MACOM) During a Fiscal Year.

$P_{ii+1}$  = Probability of Promotion From Skill Level  $i$  to Skill Level  $i+1$  and PCS Move From Current State (Skill Level & MACOM) to Another State (Higher Skill Level & Different MACOM) During a Fiscal Year.

Figure 3. Typical Submatrix



contain zero probabilities. Therefore, the transition matrix in this case only models intra-MACOM transitions. Figure 1 is a schematic of this type of model.

Modeling method two optimally distributes only the new recruits out of AIT and accounts for all PCS moves and other inter-MACOM transitions through the forecasting module by using positive transition probabilities in the off-diagonal sub-matrices. Figure 4 is a schematic of this type of model. Figure 4 shows that upon graduation from OSUT or BIT and AIT, a soldier is distributed to one of the MACOM's. When his tour of duty at that MACOM is complete and if his current enlistment is also complete he will either conduct an ETS move or re-enlist and conduct a PCS move to another MACOM. The inter-MACOM arcs in Figure 4 represent the positive off-diagonal sub-matrix transition probabilities. The arcs to the retirement and ETS sink represent attritions. Both modeling methods will be examined in this thesis and the results compared.

#### 4. The Readiness Indicator Module

The last of the four modules to be discussed in this Section is the readiness indicator module. The purpose of this module is to assign a C-rating to all the MACOM's whose personnel inventories have just been forecast. This module calculates the two ratios: percent personnel fill (PPF) and senior grade fill (SGF) for each MACOM. Since this model will only forecast enlisted end strengths, the definitions



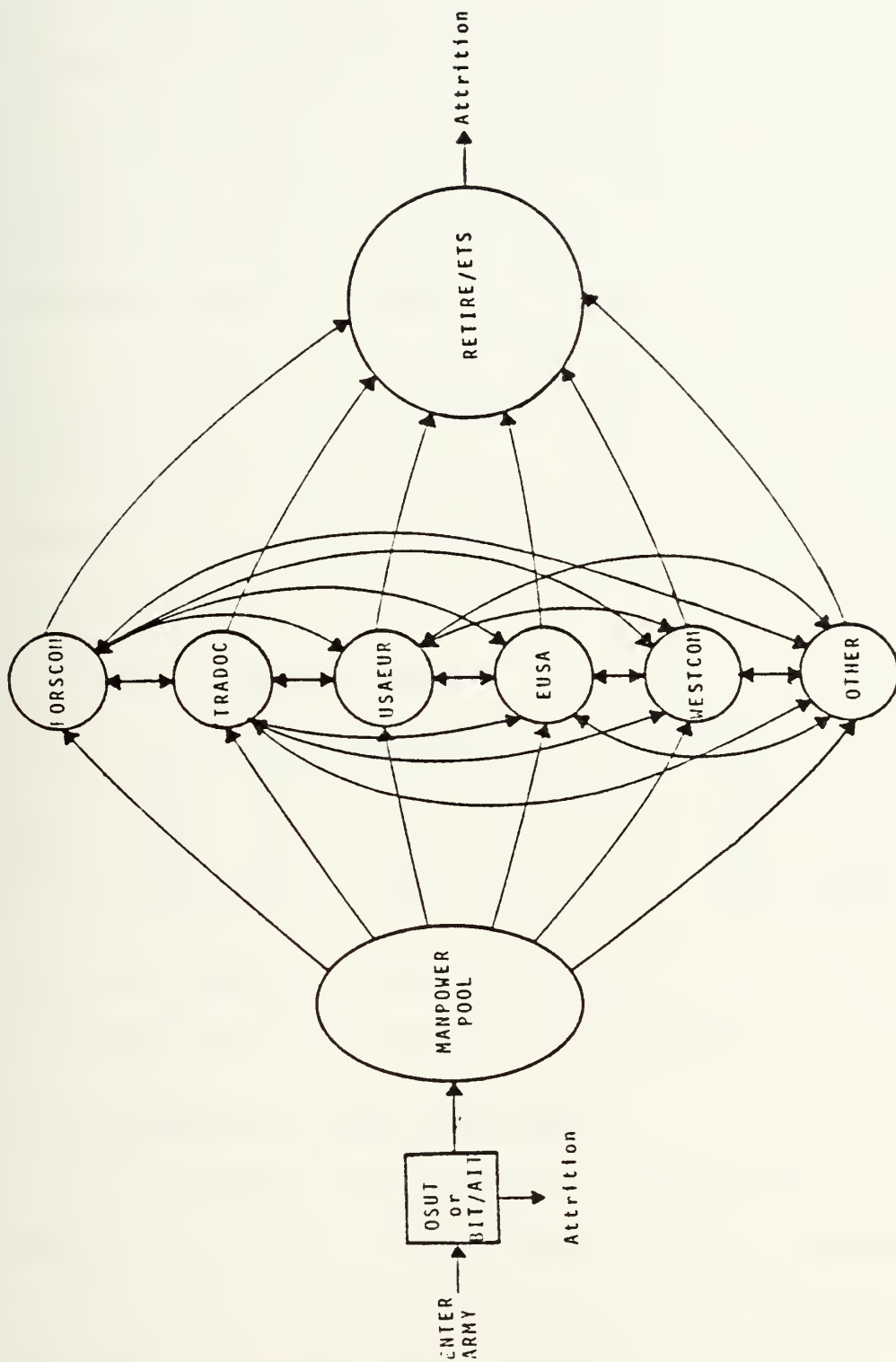


Figure 4. Modeling Method Two





of PPF and SGF will be repeated to clarify any subtle differences from the definitions of AR 220-1 stated in Chapter I. PPF is the projected total enlisted end strength of a MACOM divided by the projected total authorized number of enlisted personnel. SGF is the projected number of NCO's (skill levels 2 through 5) in a MACOM divided by the projected authorized number of NCO's.

To obtain the highest rating of C1, as established by AR 220-1, a given MACOM must meet the following criterion:  $PPF \geq .9$  and  $SGF \geq .85$ . The following tables summarize the criteria established for all the C-ratings as per AR 220-1.

Table I PPF Criteria

PPF	C-Rating
$PPF \geq .90$	C1
$.80 \leq PPF < .90$	C2
$.70 \leq PPF < .80$	C3
$PPF < .70$	C4

Table II SGF Criteria

SGF	C-Rating
$SGF \geq .85$	C1
$.75 \leq SGF < .85$	C2
$.65 \leq SGF < .75$	C3
$SGF < .65$	C4

The given MACOM's overall projected C-rating for personnel is determined by the lower of the two ratings, PPF or SGF.

#### D. DISCUSSION OF SOME ASSUMPTIONS

This Section is devoted to stating some additional assumptions, not previously stated, that are needed for the formulation of this model. First, it is assumed that personnel fill and senior grade fill are in fact good indicators of unit readiness based on personnel. No attempt



will be made to derive a better indicator of personnel readiness. Current MILPERCEN policies are not altered or changed in any way. Thus, the previously mentioned percent MOS fill is not considered as it is a local command option to use a soldier with a given MOS to fill a billet requiring a different MOS. MILPERCEN currently distributes soldiers by grade and MOS to MACOM level only. Therefore all calculations and predictions are for MACOM's only. To make predictions of future personnel strengths at the battalion or brigade level would be inappropriate as distribution data below MACOM level is either unavailable or at least very difficult to obtain.

Another aspect to MOS qualification needs to be discussed. Currently, AR 220-1 allows a command to consider a soldier MOS qualified for his position if he is one or two pay grades below, of equal pay grade, or one pay grade above the required pay grade for the billet and of identical MOS with the required billet. Again, it is a local command option to use a soldier with a particular pay grade or skill level to fill a billet requiring a different pay grade or skill level. Therefore, as in the case of the MOS mismatch, this aspect of the percent MOS fill criterion is not considered either.

Since it is known that the transition probabilities discussed in Section C vary from one MOS to another, it



would be inappropriate to develop a single transition matrix for all the MOS's in the Army. For this model to work globally, there would have to be as many transition matrices developed as there are MOS's. This thesis is attempting to develop a methodology for predicting future personnel readiness based on proven mathematical optimization and stochastic techniques. Therefore, as previously stated, it is assumed that if the methodology developed is appropriate for the MOS 11B, then it may be expanded and used to predict readiness for all MOS's.



### III. DATA

This model as currently developed is viewed as four separate modules in which the output of one module becomes the input to another. If all four modules were incorporated into one software package, the data requirements for the user would be greatly reduced. In this case, the number of civilian entrants into OSUT or BIT and current stock data for each MACOM by skill level are required for the initial fiscal year. For future fiscal years, the only data needed is the projected number of civilian entrants to OSUT or BIT.

Since this model is currently four distinct modules, it is thought appropriate that data should be specified for each module separately. The remainder of this chapter lists specific data needed for each module.

Also, it should be pointed out that data needed for the successful operation of the model by a user is different from that needed by an analyst when building or updating the model. The process of building the model or revising the model's parameters requires a great deal more raw data than is necessary for operating the model once it has been established, and it is likely that periodic revisions of the model's parameters will be necessary by an analyst. For this reason, required data for the model is stated separately for the analyst and the user. For clarity, when





some data is required by both user and analyst, it is listed in both sections. The following sections define the data requirements necessary for the four modules.

## A. THE RECRUITMENT MODULE

### 1. User Requirements

The following data is required:

- a. the number of civilians who enter 11B OSUT schools in the fiscal year prior to the initial year
- b. the number of civilians who enter 11B BIT schools in the same time frame.

It should be noted that the civilians who enter these schools are counted only if their classes commence and finish in the same fiscal year.

### 2. Analyst Requirements

The following data is required:

- a. the number of civilians who enter 11B OSUT or 11B BIT schools, desirable for as many classes as possible regardless of start and finish dates
- b. the number of skill level one soldier graduates from the above mentioned classes.

The analyst uses the above data to estimate the course completion rate. A technique for estimating the course completion rate is discussed in Chapter IV.

## B. THE DISTRIBUTION MODULE

As previously mentioned, this module optimally distributes all newly graduated skill level one soldiers and



perhaps also those who make inter-MACOM PCS moves during the fiscal year. Occasionally, there are also gains to the Army of soldiers who did not attend OSUT or BIT and AIT and who were not members of the Army in MOS 11B in the last fiscal year. Examples of such gains are prior service soldiers who re-enter active duty and soldiers who change their MOS. It is possible for a soldier in this category to enter the 11B system in any of the five skill levels. If this occurs, the soldier must be accounted for and added to the respective skill level numbers to be distributed.

#### 1. User Requirements

The user requires the following data:

- a. the total number of newly graduated skill level one soldiers from 11B OSUT or 11B AIT (provided by the recruitment module)
- b. the total number of 11B occasional gains by skill level
- c. the total number of 11B soldiers conducting a PCS move by skill level (modeling method one only, see note below).

If modeling method one is used, data requirement c. is provided by the forecasting module. A technique to account for these soldiers will be discussed in Chapter IV. If modeling method two is used, data requirement c. is not needed as PCS moves are accounted for in the forecasting module.



## 2. Analyst Requirements

The analyst requires the following data:

- a. the total number of newly graduated skill level one soldiers from 11B OSUT or 11B AIT (provided by the recruitment module)
- b. the total number of 11B occasional gains by skill level
- c. the total number of 11B soldiers conducting a PCS move by skill level (modeling method one only, see note below)
- d. upper and lower bounds on percentage fill requirements set by Army policy for each MACOM
- e. the authorized number of 11B soldiers in each MACOM by skill level
- f. the current number of 11B soldiers in each MACOM by skill level (initial year only).

Comments for data requirement c. are the same as previously discussed in the user requirements section above. Data requirement f. is obtained by observing the number of 11B soldiers in each MACOM by skill level on 1 October of the first fiscal year in question. Requirement f. for all subsequent years is provided by the forecasting module. Requirement f. is known as current stock data and all future reference to this type data will be referred to as such.

## C. THE FORECASTING MODULE

### 1. User Requirements

The user requires the following data:

- a. current stock data for each MACOM and skill level (initial year only).



This same data is required by the distribution module. All stock data for subsequent years is furnished by this forecasting module.

## 2. Analyst Requirements

The analyst requires the following data for each MACOM and skill level:

- a. the total number of 11B soldiers assigned (provided by distribution module)
- b. stock data for initial year only
- c. the total number of 11B soldiers who conduct an ETS move, retire, or die in a fiscal year
- d. the total number of 11B soldiers who were promoted and remained in their current MACOM for a fiscal year
- e. the total number of 11B soldiers who remained in their current skill level and MACOM for a fiscal year
- f. the total number of 11B soldiers who conducted an inter-MACOM PCS move in a fiscal year
- g. the total number of 11B soldiers who conducted an inter-MACOM PCS move and were promoted in a fiscal year.

Data requirement a. is provided by the distribution module and is called recruitment data. Data requirement b. for subsequent years is provided by this module. Requirement c. is called attrition flow data. Requirements d. and e. are intra-MACOM flows and are respectively categorized as promotees and stayers for each MACOM and skill level. Requirements f. and g. are inter-MACOM flows





and are respectively categorized as PCS data and PCS and promotion data for each MACOM and skill level.

There are thirty possible PCS move combinations for each skill level. Likewise, there are thirty possible combinations of PCS and promotion moves for each of the lower four skill levels.

### 3. Explanation and Examples

One can think of the stock data, mentioned above, as a complete muster of all 11B soldiers by skill level and MACOM. Each MACOM then categorizes their soldiers by skill level. The timing of this muster is important. All soldiers must be accounted for and categorized on the first day of each fiscal year, i.e., 1 October.

It is necessary to state, at this time, that not all of the flow data mentioned above is required. For instance, if all intra-MACOM flows and inter-MACOM flows are obtained, one can compute the attrition flows from the inter and intra MACOM flows and the stocks at the beginning of the fiscal year. All flow data requirements were stated as necessary to facilitate checks for consistency.

It is important to note, also, that a soldier is counted in one of the above mentioned flows if and only if he was counted as part of the unit's stocks on the first day of the fiscal year. For example, if a given soldier was promoted from skill level one to skill level two on 15 June 1980, he would be counted as a promotion flow only if he was



a skill level one member of this MACOM on 1 October 1979 and a skill level two soldier of the same MACOM on 1 October 1980. Soldiers are classified as attritions if they were members of a MACOM on 1 October 1979 and they conducted an ETS move, retired, or died prior to 1 October 1980.

Inter-MACOM flow data is best described by way of an example. Consider the two MACOM's FORSCOM and TRADOC. All 11B soldiers who were classified as skill level one soldiers in FORSCOM on 1 October 1979 and who conducted a PCS move to TRADOC and remained skill level one soldiers on 1 October 1980 are classified as making a PCS transition from FORSCOM to TRADOC and therefore will contribute to estimating that flow rate. Further, those soldiers who leave FORSCOM during the fiscal year as skill level one and arrive at TRADOC as skill level two or are promoted to skill level two at TRADOC before 1 October 1980, are classified as part of the PCS and promotion flow. Finally, soldiers who are promoted at FORSCOM from skill level one to skill level two and then conduct a PCS move to TRADOC, arriving before 1 October 1980, are also included in the PCS and promotion flow.

#### D. THE READINESS INDICATOR MODULE

##### 1. User Requirements

There are no user data requirements for this module.



## 2. Analyst Requirements

The analyst requires the following data:

- a. the authorized number of 11B soldiers for each MACOM by skill level
- b. the projected authorized number of 11B soldiers for each MACOM by skill level (see note below).

Data requirement a. is also required for the distribution module. Data requirement b. is only a requirement if it is known that authorized manning levels will change in the fiscal year for which a forecast is made.



#### IV. METHODOLOGY

This Chapter is devoted to the mathematical details of the model. In the first section the terminology and notation to be used throughout this chapter are presented. The next four sections are devoted to deriving and defining the mathematical details of the four modules.

##### A. TERMINOLOGY AND NOTATION

Throughout this thesis, lower case underscored letters refer to vectors. The underscored letters n and r respectively denote the stock and recruitment vectors. An underscored capital letter refers to a matrix. Specifically, the transition matrix is represented by P. The argument  $t$  usually within parentheses refers to the beginning of a fiscal year. The subscript  $k$  stands for an integer between one and six representing respectively the MACOM's: FORSCOM, TRADOC, USAEUR, EUSA, WESTCOM, and OTHER. Finally, the lower case letter  $p$  refers to a transition probability. For example,  $p_{ij}$  refers to the probability of transitioning from state  $i$  to state  $j$ . The states of this model are defined in Section D when the forecasting module is discussed. Notation for variables and constants peculiar to the individual modules are stated in the appropriate section.





## B. THE RECRUITMENT MODULE

### 1. Variables and Constants

The variables  $N$ ,  $S$ , and  $\alpha$  are used in this module.  $N(t)$  denotes the number of civilians entering 11B OSUT or BIT during fiscal year  $t$ .  $S(t)$  denotes the number of soldiers who graduate from 11B OSUT or AIT during the same fiscal year  $t$ . The course completion rate or its estimate is denoted by  $\alpha$ .

### 2. Estimation Of Course Completion Rate, $\alpha$

The course completion rate,  $\alpha$ , is estimated by the ratio  $S(t)/N(t)$ . Ideally, should not vary significantly from year to year. If it is found that  $\alpha$  has not significantly changed during the fiscal years 1979 to 1982, then a method of estimating the course completion rate for that period is

$$\alpha = \frac{1/4[S(79)+S(80)+S(81)+S(82)] \text{ average no. to graduate}}{1/4[N(79)+N(80)+N(81)+N(82)] \text{ average no. of entrants}}$$

If the ratio  $S(t)/N(t)$  for fiscal year 1983 is significantly different from those computed for fiscal years 1979 through 1982 and the change is thought to have been caused by a recent change in policy at HQDA, then it is reasonable to assume that the 1983 rate should be used for future forecasting purposes since it represents the new policy better than the rate based on the average numbers of all the years 1979-1983.



### 3. Forecasting Recruitment

To forecast the total number of skill level one 11B graduates during FY 1984, one multiplies the projected number of entrants during FY 1984,  $N(84)$ , by the estimated course completion rate,  $\alpha$ , to obtain the value:

$$S(t) = \alpha N(t). \quad (2)$$

#### C. THE DISTRIBUTION MODULE

As may be recalled from Chapter II, the distribution module is a linear program with the objective of maximizing the minimum level of either PPF or SGF for each MACOM subject to personnel fill criteria established by the Department of the Army. This section gives details of the linear program and states the two minor modifications needed for the two previously mentioned methods of modeling the inter-MACOM transitions.

##### 1. The Linear Program

For each MACOM there is an overall percent personnel fill criterion and a senior grade fill criterion. The objective is to allocate soldiers such that over all MACOM's the smallest of these criteria is maximized. This objective function is subject to four linear constraints. The first constraint states that the sum of all soldiers with MOS 11B distributed to all MACOM's in question cannot exceed the total number of soldiers with that skill level available for distribution. The second and third constraints state that



the percentage fill must lie between upper and lower bounds established by Army policy for that MACOM. The last constraint is a non-negativity constraint on the number of soldiers allocated. The following is the mathematical formulation of the linear program of the distribution module.

a. Variable Definitions

$T_j$  = maximum number of soldiers to distribute with skill level  $j$

$M_k$  = upper bound (percentage fill) for MACOM  $k$

$m_k$  = lower bound (percentage fill) for MACOM  $k$

$R_{jk}$  = authorized number of soldiers with skill level  $j$  in MACOM  $k$

$R_k$  = total authorized number of soldiers in MACOM  $k$

$n_{jk}$  = current number of soldiers with skill level  $j$  in MACOM  $k$

$c_k$  =  $1/R_k$

$b_k$  =  $\sum_j n_{jk}/R_k$

$c'_k$  =  $1 / \sum_{j=2}^5 R_{jk}$

$b'_k$  =  $\sum_{j=2}^5 n_{jk} / \sum_{j=2}^5 R_{jk}$

$x_{jk}$  = decision variable, the number of soldiers with skill level  $j$  to distribute to MACOM  $k$ .



## b. Problem Formulation

$$\text{MAX}_{x_{j,k}} \left[ \text{MIN}_{j,k} \left( \sum_j (n_{jk} + x_{jk}) / R_k, \sum_{j=2}^5 (n_{jk} + x_{jk}) / \sum_{j=2}^5 R_{jk} \right) \right]$$

$$\begin{aligned} \text{Subject to:} \quad & \sum_k x_{jk} \leq T_j \\ & \sum_j n_{jk} + \sum_j x_{jk} \leq M_k \\ & \sum_j n_{jk} + \sum_j x_{jk} \geq m_k \\ & x_{j,k} \geq 0 \end{aligned}$$

The above formulation is equivalent to the following:

$$\begin{aligned} & \text{MAX } x_0 \\ \text{Subject to} \quad & x_0 \leq c_k \sum_j x_{jk} + b_k \quad k=1,2,3,4,5,6 \\ & x_0 \leq c'_k \sum_{j=2}^5 x_{jk} + b'_k \quad k=1,2,3,4,5,6 \\ & \sum_k x_{jk} \leq T_j \quad j=1,2,3,4,5 \\ & \sum_j x_{jk} \leq M_k R_k - \sum_j n_{jk} \quad k=1,2,3,4,5,6 \\ & \sum_j x_{jk} \geq m_k R_k - \sum_j n_{jk} \quad k=1,2,3,4,5,6 \\ & x_{j,k} \geq 0 \quad \forall j,k \end{aligned}$$

Using the above formulation not only insures an optimal distribution of soldiers but it also provides the recruitment vector input to the forecasting module.





## 2. Distribution Using Modeling Method One

Recall from Chapter II that modeling method one requires the distribution module to optimally distribute all 11B soldiers newly graduated from OSUT and AIT, all occasional 11B gains, and all inter-MACOM transitions. This modification requires the definition of the following variables:

- a.  $S_1$  represents the number of newly graduated 11B soldiers from OSUT or AIT in a fiscal year
- b.  $G_j$  represents the number of occasional 11B gains of skill level  $j$  in a fiscal year
- c.  $PCS_j$  represents the number of 11B soldiers with skill level  $j$  conducting an inter-MACOM PCS move plus the number of 11B soldiers with skill level  $j-1$  who conduct an inter-MACOM PCS move combined with promotion to skill level  $j$  during a fiscal year.  $PCS_j$  is provided by the forecasting module when using modeling method one.
- d.  $T_1 = S_1 + G_1 + PCS_1$
- e.  $T_j = G_j + PCS_j \quad j = 2, 3, 4, 5.$

## 3. Distribution Using Modeling Method Two

Modeling method two requires the distribution module to distribute optimally only those 11B soldiers newly graduated from OSUT and AIT and the occasional 11B gains. Thus, when using modeling method two,  $T_j$  must be modified in the following manner:

- a.  $T_1 = S_1 + G_1$
- b.  $T_j = G_j \quad j = 2, 3, 4, 5.$



#### 4. Modification Due to the TTHS Account

Upon receipt of the data for the total authorized number,  $R_k$ , of soldiers in each MACOM and the authorized number of soldiers,  $R_{jk}$ , in each MACOM by skill level, a large discrepancy was noted between the authorized numbers and the assigned numbers indicated by the stock data. The largest discrepancy in all MACOM's was in skill level one. The most significant difference was noted in TRADOC's skill level one due to the large number of soldiers attending OSUT or BIT and AIT. It was determined that all stock and flow data had consistently included soldiers who were in the TTHS account.

The TTHS account data was then obtained for all fiscal years concerned. Although ideally it would be more appropriate to exclude all soldiers in the TTHS account from the stock and flow data, it was reasoned that if the number of soldiers in the TTHS account on the first day of each fiscal year concerned was added to the authorized number of soldiers for each MACOM by skill level set by Army policy, then the true personnel shortages would be reflected on comparison with the similarly inflated stock data.

For this reason, the total number of soldiers in the TTHS account by skill level and MACOM is added to the total number of soldiers authorized by skill level and MACOM to yield  $R_{jk}$ . Similarly, the total number of soldiers in the TTHS account by MACOM is added to the total authorized



number of soldiers in each MACOM to yield  $R_k$ . This procedure will be analyzed more in Chapter V.

### 5. Formation of Recruitment Vector, $r$

The linear program of the distribution module assigns numbers to the thirty decision variables,  $x_{jk}$ . These decision variables correspond to the numbers of soldiers to be distributed or allocated to each of the six MACOM's in each of the five skill levels. To form the recruitment vector  $\underline{r}(t)$ , the decision variables are ordered by skill level for each MACOM 1 to 6. The structure of this module's recruitment vector for fiscal year  $t$  is shown in Figure 5 below.

$$\underline{r}(t) = (\overbrace{x_{11}, x_{21}, x_{31}, x_{41}, x_{51}}^{\text{FORSCOM}}, \overbrace{x_{12}, x_{22}, x_{32}, x_{42}, x_{52}}^{\text{TRADOC}}, \dots, \overbrace{x_{16}, x_{26}, x_{36}, x_{46}, x_{56}}^{\text{OTHER}})$$

Figure 5. Recruitment Vector

### D. THE FORECASTING MODULE

This section describes the formation of vectors and the estimation of parameters needed for this module. The formation of the transition matrix,  $\underline{P}$ , is also discussed. A technique to account for inter-MACOM transitions using an expanded transition matrix is also shown. Techniques for estimating necessary parameters are described. The actual



technique used for this modeling effort is discussed in Chapter V.

Prior to a discussion of the above mentioned techniques, a definition of states is needed. The states of this system are defined by the skill levels and the MACOM's of the model. States 1 through 5 correspond to the five skill levels in FORSCOM, states 6 through 10 correspond to the five skill levels in TRADOC, states 11 through 15 correspond to the five skill levels in USAEUR, states 16 through 20 correspond to the five skill levels in EUSA, states 21 through 25 correspond to the five skill levels in WESTCOM, and finally states 26 through 30 correspond to the five skill levels in OTHER. A summary of this list is given in Table 3.

#### 1. Formation of the Stock Vector, $n$

The formation of the stock vector is analogous to the formation of the recruitment vector described above. The stocks are the numbers of soldiers in MACOM  $k$  with skill level  $j$ . Let  $n_{jk}(t)$  represent the number of soldiers with skill level  $j$  in MACOM  $k$  on 1 October of fiscal year  $t$ . To form the stock vector  $n(t)$  the stocks are ordered by skill level for each MACOM 1 to 6. The structure of this module's stock vector for fiscal year  $t$  is shown in Figure 6 below.

#### 2. Estimation of Parameters

Let  $n_{ij}(t-1, t)$  represent the number of soldiers transitioning from state  $i$  to state  $j$  during a fiscal year.





Table III. States of the System

<u>STATE</u>	<u>MEANING</u>
1	SL1 FORSCOM
2	SL2 FORSCOM
3	SL3 FORSCOM
4	SL4 FORSCOM
5	SL5 FORSCOM
6	SL1 TRADOC
7	SL2 TRADOC
8	SL3 TRADOC
9	SL4 TRADOC
10	SL5 TRADOC
11	SL1 USAEUR
12	SL2 USAEUR
13	SL3 USAEUR
14	SL4 USAEUR
15	SL5 USAEUR
16	SL1 EUSA
17	SL2 EUSA
18	SL3 EUSA
19	SL4 EUSA
20	SL5 EUSA
21	SL1 WESTCOM
22	SL2 WESTCOM
23	SL3 WESTCOM
24	SL4 WESTCOM
25	SL5 WESTCOM
26	SL1 OTHER
27	SL2 OTHER
28	SL3 OTHER
29	SL4 OTHER
30	SL5 OTHER

$$\mathbf{x}(t) = (\overbrace{x_{11}, x_{21}, x_{31}, x_{41}, x_{51}}^{\text{FORSCOM}}, \overbrace{x_{12}, x_{22}, x_{32}, x_{42}, x_{52}}^{\text{TRADOC}}, \dots, \overbrace{x_{16}, x_{26}, x_{36}, x_{46}, x_{56}}^{\text{OTHER}})$$

Figure 6. Stock Vector



Here the subscripts  $i$  and  $j$  refer to the states  $i$  and  $j$  and the arguments  $t-1$  and  $t$  refer to the beginning and the end of the fiscal year in which the flow occurred. The relationship between the notation used in the last section and the notation used in this section is the following: the state represented by  $i$  is such that

$$i = 5(k-1) + j,$$

where  $j$  and  $k$  respectively represent skill level and MACOM.

By way of example, the number of soldiers transitioning from skill level three ( $j = 3$ ) in USAEUR ( $k = 3$ ) to skill level four ( $j = 4$ ) in FORSCOM ( $k = 1$ ) during fiscal year  $t - 1$  to  $t$  is the same as the number of soldiers transitioning from state 13 to state 4 during that fiscal year. Thus, the notation  $n_{13,4}(t-1,t)$  is used.

#### a. Explanation of Transition Probabilities

The basic method of estimating the probability,  $p_{ij}$ , of transitioning from state  $i$  to state  $j$  is given by the ratio:

$$p_{ij} = \frac{n_{ij}(t-1, t)}{n_i(t-1)} \quad (3)$$

where  $n_i(t-1)$  represents the number of soldiers in state  $i$  at time period  $t-1$ . [Ref. 2: p. 105]

Once calculated, the transition probability,  $p_{ij}$ , is placed in the transition matrix,  $\underline{P}$ , in row  $i$  and column  $j$ . Figure 7 shows the transition matrix with the rows and columns numbered for clarity.



	FORSCOM					TRADOC					USAEUR					EUSA					WESTCOM					OTHER				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	1																													
	2																													
FORSCOM	3												$P_{3,13}$																	
	4				$F_{45}$																									
	5																													
	6																													
	7																													
TRADOC	8																													
	9																													
	10																													
	11																									$F_{11,27}$				
	12																													
USAEUR	13																													
	14																													
	15																													
	16															$P_{16,16}$														
	17																													
EUSA	18																													
	19																													
	20																													
	21																													
	22																													
WESTCOM	23																													
	24																													
	25																													
	26																													
	27																													
OTHER	28																													
	29																													
	30																													

Figure 7. The Transition Matrix (With Details)



b. Examples

The following four examples clarify the procedures of estimating the parameters  $p_{ij}$ . The first two explain intra-MACOM transitions and the last two explain inter-MACOM transitions. First, the number of soldiers promoted from skill level four to skill level five during a fiscal year and who start and finish the fiscal year in FORSCOM is denoted as  $n_{45}(t-1, t)$ . The corresponding promotion rate within FORSCOM is estimated as:

$$p_{45} = \frac{n_{45}(t-1, t)}{n_4(t-1)} .$$

Second, the number of soldiers remaining in skill level one in EUSA during a fiscal year is denoted as  $n_{16,16}(t-1, t)$  and the staying rate within that skill level in EUSA is estimated as:

$$p_{16,16} = \frac{n_{16,16}(t-1, t)}{n_{16}(t-1)} .$$

Third, the number of skill level three soldiers conducting a PCS move from FORSCOM to USAEUR during a fiscal year is denoted as  $n_{3,13}(t-1, t)$  and the corresponding PCS rate is estimated as:

$$p_{3,13} = \frac{n_{3,13}(t-1, t)}{n_3(t-1)} .$$

Finally, the number of skill level one soldiers promoted to skill level two and conducting a PCS move from USAEUR to





OTHER during a fiscal year is denoted as  $n_{11,27}(t-1, t)$  and the corresponding PCS and promotion rate is estimated as:

$$P_{11,27} = \frac{n_{11,27}(t-1, t)}{n_{11}(t-1)} .$$

The position of the four transition probabilities are shown in Figure 7.

### c. Additional Estimation Techniques

It is best to collect stock and flow data, as described in Chapter III, for several fiscal years. If the individual  $p_{ij}$ 's corresponding to different fiscal years are nearly equal, it is reasonable to estimate the  $p_{ij}$ 's by calculating average rates in the following manner:

$$p_{ij} = \frac{\sum_t n_{ij}(t-1, t)}{\sum_t n_i(t-1)} , \quad (4)$$

where the summations in the numerator and denominator extend over all the fiscal years for which data is available.

[Ref. 2: p. 105]

As an aid to determine if individual  $p_{ij}$ 's corresponding to different fiscal years are nearly equal, one may graph the calculated transition probability estimates on the vertical axis versus the fiscal years on the horizontal axis for as many years as possible. By connecting all plotted probabilities with straight line segments, one can easily determine outlier years if any. Additional research may indicate that an outlier fiscal year



in question may have, in fact, been abnormal. If this is the case, then the analyst is justified in excluding that particular fiscal year's data in his calculation of  $p_{ij}$ . [Ref. 2: p. 106]

If the individual  $p_{ij}$ 's for all years concerned do not vary significantly and the average rate is chosen using equation 4 above, then an average standard error of the estimate may be computed using the following equation:

$$\left[ p_{ij}(1-p_{ij})/Y^{-1} \sum_t n_i(t-1) \right]^{1/2}, \quad (5)$$

where  $Y$  is the number of years of data. This estimate of the error assumes a binomial model in which all soldiers behave independently and are subject to the same transition rates. From equation 5, one can graph an error band, of width two times the above standard error, around the previously plotted  $p_{ij}$ 's discussed above. If the majority of the  $p_{ij}$ 's fall within the error band, then one can be reasonably confident that there are no systemic trends to change the calculated rates. Likewise there is no evidence to suggest that the rates have changed appreciably over the fiscal years examined. [Ref. 2: p. 129]

Examples of the techniques used to determine the transition probabilities for this thesis are included in the analysis section of Chapter V.



#### d. Estimation of Attrition Rates

Attrition is categorized as such by the state from which the attrition occurs. Thus, the notation  $w_i$  indicates the probability of a soldier conducting an ETS move or retiring from state  $i$ . Let the number of soldiers leaving the Army from state  $i$  during the fiscal year  $t-1$  to  $t$  be denoted as  $n_{i, s+1}(t-1, t)$  where  $s$  is the number of states in the system. The probability of attrition,  $w_i$ , from state  $i$  is then estimated as:

$$w_i = \frac{n_{i, s+1}(t-1, t)}{n_i(t-1)} \quad . \quad (6)$$

[Ref. 2: p. 4]

When the attrition rates,  $w_i$ , are estimated for all the states, they are used to form the attrition vector. The attrition vector is a column vector and is illustrated in Figure 7 adjacent to the transition matrix.

Referring once again to Figure 7, note that since each soldier must either stay where he is, transition to another state, or attrit from the Army, the following mathematical identity is valid:

$$\sum_{j=1}^{30} p_{ij} + w_i = 1 \quad i = 1, 2, 3, \dots, 30 \quad (7)$$

The above relationship is useful as a check for consistency among the stock and flow data. [Ref. 2: p. 87]



### 3. Formation of the Transition Matrix, P

Modeling method two requires a transition matrix exactly like the one described thus far as the forecasting module inherently accounts for all inter-MACOM transitions. However in modeling method one, a technique must be developed to account for all the inter-MACOM transitions so that these transitions may be recycled back to the distribution module. The most efficient technique to account for these inter-MACOM transitions appears to be one of expanding the transition matrix.

Development of the standard transition matrix for modeling method two will be discussed first. This will be followed by a description of the expanded matrix needed for modeling method one.

#### a. Modeling Method Two

Once all intra-MACOM transition probabilities have been estimated, the six main diagonal sub-matrices can be formed as shown in Figure 3. Similarly, if all inter-MACOM transition probabilities are estimated the thirty off-diagonal sub-matrices are formed also as shown in Figure 3. The main-diagonal and off-diagonal matrices are then embedded in the transition matrix in the sequence shown in Figure 2.

#### b. Modeling Method One

It may be recalled from Figure 1, that all six MACOM's transition soldiers to a PCS pool. The PCS pool





then sends these soldiers back to the manpower pool for distribution. To accomplish the goal of accounting for the inter-MACOM transitions, the PCS pool as depicted in Figure 1 is considered to be a seventh MACOM. However, the duration of service for soldiers in this MACOM is considered to be instantaneous.

The presence of a seventh MACOM in the system necessitates the expansion of the transition matrix by five rows and five columns. The seventh "MACOM" in the expanded transition matrix is referred to as "POOL". Figure 8 is a schematic of the expanded transition matrix.

The current stock vector,  $\underline{n}(t-1)$ , the future stock vector,  $\underline{n}(t)$ , and the recruitment vector,  $\underline{r}(t)$ , also need to be expanded by five more elements. Initially the five new components of the current stock vector are all zeros because it is assumed that there are no soldiers left currently undistributed in the pool. The five new components of the recruitment vector are all zeros for the same reason. Positions thirty-one through thirty-five of the future stock vector will be the skill levels one through five for the 11B soldiers to be optimally distributed through the distribution module. The numbers in the last five positions of the future stock vector are projections of the numbers of inter-MACOM transitions that will occur as estimated by the Markov Chain equation (equation (1)). When passed to the distribution module, these numbers become the



	FORSCOM	TRADOC	USAEUR	EUSA	WESTCOM	OTHER	POOL
FORSCOM	FORSCOM SUB-MATRIX						FROM FORSCOM INTER-MACOM TRANSITIONS
TRADOC		TRADOC SUB-MATRIX					FROM TRADOC INTER-MACOM TRANSITIONS
USAEUR			USAEUR SUB-MATRIX				FROM USAEUR INTER-MACOM TRANSITIONS
EUSA				EUSA SUB-MATRIX			FROM EUSA INTER-MACOM TRANSITIONS
WESTCOM					WESTCOM SUB-MATRIX		FROM WESTCOM INTER-MACOM TRANSITIONS
OTHER						OTHER SUB-MATRIX	FROM OTHER INTER-MACOM TRANSITIONS
POOL							

Figure 8. Expanded Transition Matrix



variable  $PCS_j$  previously defined when discussing the distribution module.

The first six sub-matrices in the last five rows of the expanded transition matrix contain all zero elements. In order to have positive transition probabilities in these elements the possibility would have to exist that a soldier could remain in the POOL from one fiscal year to the next. The assumption of instantaneous transition negates this possibility.

The first six sub-matrices in the last five columns of the expanded transition matrix are composed of transition probabilities that are estimated in the same manner as described in the previous section. The main-diagonal elements in these sub-matrices indicate the probability of a soldier conducting a PCS move from one of the MACOM's to another MACOM while staying in his skill level. The above diagonal elements indicate the probability that a soldier conducts a PCS move to another MACOM and is promoted in the same fiscal year.

Finally, it seems conceivable to obtain positive transition probabilities in the sub-matrix located in the lower right hand corner of the expanded  $\underline{P}$  matrix as transitions within this "PCS POOL". For example, some hospitalized soldiers and some prisoners do remain in that status for more than one fiscal year. However, the inclusion of this last category is not an attempt to capture



the dynamics of the TTHS account, but rather it is a means of accounting for inter-MACOM transitions. The TTHS account is assumed to be of negligible size in the forecasting module. Thus, the assumption of a null sub-matrix is appropriate here as well.

#### E. THE READINESS INDICATOR MODULE

The readiness indicator module assigns a C-rating for each MACOM based upon the two ratios PPF and SGF. The ratios PPF and SGF are as follows:

$$PF_k = 1/R_k \sum_{j=1}^5 n_{jk} \quad k = 1, 2, 3, 4, 5, 6 \quad (8)$$

$$SGF_k = \frac{\sum_{j=2}^5 n_{jk}}{\sum_{j=2}^5 R_{jk}} \quad k = 1, 2, 3, 4, 5, 6 \quad (9)$$

Note that the numbers  $R_k$  and  $R_{jk}$  are the modified numbers due to inflation by the TTHS account mentioned in the section on the distribution module. The C-rating assigned is dictated by the criteria discussed in Chapter II and shown in Tables I and II.





## V. RESULTS

To test the model developed in Chapters II and IV, the inventories of MOS 11B soldiers of the six MACOM's considered here were forecast for 1 October 1982 starting with the inventories on 1 October 1981. This chapter is devoted to presenting and analyzing the results of this forecasting effort.

The numbers of civilian entrants into OSUT or BIT and AIT were not available and therefore an estimate of the course completion rate from the U.S. Army OSUT and BIT and AIT programs was not computed. However, the total numbers of newly graduated 11B soldiers for each fiscal year examined were available and this data is used as input to the distribution module.

The data base for this forecasting effort consists of stock and flow data from fiscal years 1979, 1980, and 1981 for all MACOM's with the exception of WESTCOM. Stock and flow data for fiscal year 1979 is unavailable for WESTCOM as it was designated a separate MACOM after 1 October 1978. Further, the total numbers of soldiers who conducted inter-MACOM transitions by skill level during the above fiscal years are available. However, the distribution of these inter-MACOM transitions among the six MACOM's was available for fiscal year 1981 only. Consequently, no true test of



modeling method two could be carried out as forecasting of 1 October 1982 inventories could be made only by using fiscal year 1982 flow data. This is shown in Section B.

In Section A the results of this model's distribution module are presented along with a comparison of actual and projected unit readiness. The results of this model's forecasting module is presented in Section B along with a comparison of actual and forecasted 1 October 1982 MACOM end strengths.

#### A. THE DISTRIBUTION MODULE

The values of previously defined parameters needed in this module are given below in Tables IV and V. All parameters whose values are given in Tables IV and V are defined in Section C of Chapter IV when discussing the linear program.

Table IV. Parameters of Distribution Module

	$R_{jk}^*$					$R_k^*$	$C_k^*$	$C_k'^*$
	SL1	SL2	SL3	SL4	SL5	TOTAL		
FORSCOM	20744	3964	2912	1604	1228	30452	$3.28 \times 10^{-5}$	$1.030 \times 10^{-4}$
TRADOC	3284	338	2771	1691	1091	9175	$1.09 \times 10^{-4}$	$1.698 \times 10^{-4}$
USAEUR	9770	2252	1324	596	409	14343	$6.97 \times 10^{-5}$	$2.187 \times 10^{-4}$
EUSA	1519	360	265	109	75	2328	$4.30 \times 10^{-3}$	$1.236 \times 10^{-3}$
WESTCOM	1662	361	215	91	63	2392	$4.18 \times 10^{-4}$	$1.270 \times 10^{-3}$
OTHER	788	174	135	141	86	1324	$7.55 \times 10^{-4}$	$1.866 \times 10^{-3}$

\* Symbol definitions are given in Section C of Chapter IV.



Table V. Additional Parameters of Distribution Module

	$\xrightarrow{\quad} n_{jk}^{**} \xleftarrow{\quad}$					$b_k^{**}$	$b_k'^{**}$	$M_k^{**}$	$m_k^{**}$
	SL1	SL2	SL3	SL4	SL5				
FORSCOM	15774	3695	2502	1454	1043	.80353	.89555	1.0	.95
TRADOC	744	615	1584	1860	881	.61951	.83857	1.0	.97
USAEUR	7381	1844	1523	587	332	.81343	.9372	1.02	.99
EUSA	1044	270	238	102	51	.73239	.81706	1.0	.98
WESTCOM	1089	320	224	102	56	.74875	.96164	1.0	.95
OTHER	630	223	181	153	89	.96375	1.20522	1.0	.95

\*\* Symbol definitions are given in Section C of Chapter IV.

The authorized number,  $R_{jk}$ , of 11B soldiers in each MACOM by skill level and the total number,  $R_k$ , of authorized 11B soldiers in each MACOM for 1 October 1982 were obtained from MILPERCEN. To obtain the values listed above in Table IV, the total numbers of 11B soldiers by MACOM and skill level who were in the TTHS account on 1 October 1982 were added to the authorized number of soldiers in each category. The reason for this was explained in Section C of Chapter IV.

Since recruitment is assumed to occur at the end of the fiscal year in the forecasting module, it is necessary to determine the inventory in the six MACOM's at the end of the fiscal year just prior to recruitment. This is accomplished by "aging" the force using the basic equation

$$\underline{n}(t) = \underline{n}(t-1) \underline{P} + \underline{r}, \quad (1)$$

with the inventory on 1 October 1981 as the current stock vector,  $\underline{n}(t-1)$ , and the recruitment vector,  $\underline{r}$ , equal to the



null vector. The resulting stock vector,  $\underline{n}(82)$ , yielded the inventories,  $n_{jk}$ , listed in Table V.

For the purpose of verifying the distribution module of modeling method two  $T_j$  is made equal to the newly graduated 11B soldiers from OSUT and AIT plus the occasional gains to skill level  $j$ . The reader will recall from Section C of Chapter IV that  $T_1 = S_1 + G_1$  and  $T_j = G_j$  for all skill levels two through five. Thus, using these relationships and the data obtained from MILPERCEN, the numbers  $T_j$  for fiscal year 1981 are as follows:

1.  $T_1 = 9262$
2.  $T_2 = 292$
3.  $T_3 = 157$
4.  $T_4 = 132$
5.  $T_5 = 123.$

The distribution module was then run to distribute the above mentioned soldiers to the six MACOM's. The output was then added to the inventory data listed in Table V to obtain the inventory data for 1 October 1982. In effect, this action uses equation (1) with this module's recruitment vector as the vector  $\underline{r}$ . Then equations (8) and (9), from Section E of Chapter IV, are used to determine PPF and SGF. Table VI displays the results.

The differences in percentage fill listed in Table VI are due to the distribution algorithm used since the same transition matrix that was used to age the force was also employed to derive the inventories needed in the calculation of the readiness indicators PPF and SGF.





Table VI. Results of Distribution Module

Actual Distribution			Model's Distribution	
	PPF	SGF	PPF	SGF
FORSCOM	.942	.932	.937(.915)	.915
TRADOC	.933	.863	.936(.915)	.915
USAEUR	.944	.966	.946(.928)	.915
EUSA	.907	.859	.957(.915)	.915
WESTCOM	.910	.978	.988(.915)	.915
OTHER	1.120	1.270	.964(.964)	.915

Although both the Army's actual distribution and this models distribution yield a rating of C1 for all MACOM's, this model's distribution algorithm clearly yields better results in terms of minimum percentage fill.

It should be mentioned that the distribution module did not distribute all skill level one soldiers. The binding constraint is on senior grade fill. The optimal percentage fill for SGF is obtained by the linear program even though not all skill level one soldiers are assigned. Thus, in the linear program the slack variable associated with skill level one soldiers is positive. In this example, 1631 skill level one soldiers were assigned to a slack variable. Since it is clearly better to assign these excess soldiers somewhere, the 1631 extra skill level soldiers were manually assigned to the six MACOM's in approximately equal proportions to obtain the results listed in Table VI. The numbers in parentheses indicate the PPF of the six MACOM's prior to the manual assignment of the 1631 extra skill level one soldiers.



The situation described above can be overcome by additional constraints such as penalty constraints on slack variables or lower bounds added to individual skill level requirements. Care must be taken to insure the equitable distribution of soldiers in excess of the binding constraint. Additional thesis research is needed to further evaluate and refine this model's distribution algorithm.

#### B. THE FORECASTING MODULE

Determination of the transition probabilities is a vital part of the forecasting module. Equation (3) from Section D of Chapter IV is used to determine the yearly intra-MACOM promotion probabilities peculiar to each MACOM. Likewise, equation (6) from the same Section is used to determine the yearly attrition rates from each state that are peculiar to each MACOM. The promotion and attrition probabilities, as calculated from the data for FY's 1979-1981 according to procedures described in Section D of Chapter IV, were plotted versus fiscal years. These graphs are presented in Appendix A.

These graphs show that although there are no outlier years among the promotion rates, they were very unstable during the fiscal years 1979 to 1981. The graphs do indicate that there is a strong correlation between the promotion rates of FORSCOM and USAEUR.



From skill level one, the promotion rates went up from FY 1979 to FY 1980 and down again in FY 1981 but not as low as their FY 1979 level. Promotion rates from skill level two increased from FY 1979 to FY 1980 to FY 1981, although the increase from FY 1980 to FY 1981 was at a slower rate. Promotion rates from skill level three changed little from FY 1979 to FY 1980, then they increased from FY 1980 to FY 1981. From skill level four the promotion rates dropped from FY 1979 to FY 1980 and then the rates went back up except in EUSA where the reverse was true and in TRADOC where the rate decreased by only a small margin in FY 1980 and FY 1981. The above statements are not valid for WESTCOM where only two data points (FY 1980 and FY 1981) were available and occasionally for OTHER.

The attrition rates from skill level one decreased in FY 1980 and again in FY 1981 for all MACOM's with the exception of TRADOC where the reverse is true. All MACOM attrition rates from skill level two decreased in FY 1980 and again in FY 1981. From skill level three all MACOM attrition rates decreased in FY 1980 and again in FY 1981 with the exception of TRADOC and EUSA where the attrition rates increased slightly in the last year. The attrition rates from skill level four from all MACOM's decreased in FY 1980 and FY 1981 with the exception of EUSA where the attrition rate in FY 1981 increased slightly. The attrition rates from skill level five increased in all MACOM's in FY 1980 and then



stayed the same or decreased slightly except in USAEUR where the reverse is true. Again WESTCOM and OTHER must be excepted on the same grounds as before.

As discussed in Chapter IV, equation (5) is used to determine the average error over this three year period. Referring once again to Appendix A the dashed horizontal lines are drawn one standard deviation above and below the average rate as computed by equation (5). Ideally the rates estimated (from the single years' data) should all lie between the two dashed lines. However, in this application, only in a few cases did two or more points lie within the error band.

Since no patterns or outlier years are indicated and since there is no available information to indicate which of the calculated transition probabilities is better than any other, equation (4) is used to compute an average transition probability over the three fiscal years for each MACOM and skill level. Appendix B contains the derived transition matrices for modeling methods one and two. Equation (4) is used to determine the rates for modeling method one and equation (3), involving a single year only, is used to determine the transition rates for modeling method two.

As stated earlier, a valid test of the forecasting technique using modeling method two is not possible since the appropriate inter-MACOM transition data for fiscal years 1979 and 1980 is unavailable at this time. The forecast





made using modeling method two is included to demonstrate the methodology. Since the transition probabilities in this case were estimated from FY 1981 data only, deviation of the projected 1 October 1982 inventory from the actual 1 October 1982 inventory should be caused by rounding errors only, unless some invalid modeling assumptions were made. Table VII indicates the results of the modeling method two forecast.

There are two projections in Table VII that indicate that an invalid assumption may have been made. Skill level one in the MACOM's WESTCOM and OTHER have errors great enough to indicate something other than rounding error is to blame. The most likely reason for the discrepancy is that demotions to skill level one in WESTCOM and OTHER are not negligible as it is assumed in this model. Some further analysis is necessary to determine if demotions are significant. If demotions are found to be significant, then additional intra and inter-MACOM transition data including demotions is needed to estimate the "demotion rates" so they can be included in the transition matrix.

Modeling method one is used to predict 1 October 1982 inventories starting from actual 1 October 1981 inventories. This run of the model is in fact a true test of the forecasting technique described in this thesis as the parameters were determined from three previous years' data. Table VIII indicates the results of this forecast.



Table VII. Results of Modeling Method Two Forecast

	Skill Level	1 OCT 82 Actual Inventory	1 OCT 82 Projected Inventory	Error
FORSCOM	1	19710	19642	.003
	2	3869	3881	.003
	3	2591	2567	.009
	4	1507	1507	0
	5	1094	1095	.0009
TRADOC	1	3485	3472	.004
	2	652	659	.011
	3	1628	1626	.001
	4	1888	1888	0
	5	911	911	0
USAEUR	1	9179	9125	.006
	2	1897	1887	.005
	3	1559	1553	.004
	4	616	616	0
	5	363	363	0
EUSA	1	1416	1416	0
	2	279	278	.004
	3	247	246	.004
	4	112	110	.018
	5	57	56	.018
WESTCOM	1	1597	1462	.085
	2	323	322	.003
	3	227	227	0
	4	106	106	0
	5	58	59	.017
OTHER	1	899	807	.102
	2	235	232	.013
	3	191	190	.005
	4	163	163	0
	5	91	91	0



Table VIII. Results of Modeling Method One Forecast

	Skill Level	1 OCT 82 Actual Inventory	1 OCT 82 Projected Inventory	Error
FORSCOM	1	19710	18035	.085
	2	3869	3517	.091
	3	2591	2173	.161
	4	1507	1376	.087
	5	1094	1046	.044
TRADOC	1	3485	3577	.026
	2	652	628	.037
	3	1628	1617	.007
	4	1888	1776	.059
	5	911	869	.046
USAEUR	1	9179	8619	.061
	2	1897	1857	.021
	3	1559	1471	.056
	4	616	567	.079
	5	363	369	.017
EUSA	1	1416	1415	.0007
	2	279	270	.032
	3	247	248	.004
	4	112	111	.009
	5	57	59	.035
WESTCOM	1	1597	1523	.046
	2	323	353	.093
	3	227	251	.106
	4	106	114	.075
	5	58	60	.034
OTHER	1	899	834	.072
	2	235	230	.021
	3	191	186	.026
	4	163	167	.025
	5	91	96	.055



It may be noted that if this model's distribution and forecasting modules had been tested together, then the reasons for the deviations of the projections from the actual inventories could not be pinpointed as readily as is the case with the separate tests. Therefore, separate validation tests were conducted for the two modules.

In Table VIII 18 of 30 projections (60 percent) have less than five percent error. Only two of the projections (6.67 percent) have more than ten percent error. The larger errors occur in skill levels one, two, and four. It is possible that two previously made assumptions are invalid. First, the assumption of negligible demotions may be invalid and second, the assumption of only one transition in a fiscal year may also be invalid. It may be quite common for a soldier to be promoted to skill level two and then conduct an ETS move in the same fiscal year. Furthermore, the skill level four errors appear in overseas MACOM's (USAEUR and WESTCOM) and FORSCOM. It is possible that skill level four soldiers in overseas MACOM's conduct a PCS move to FORSCOM just prior to retirement in order to retire in the United States. This would negate the one transition per fiscal year assumption in skill level four in these three MACOM's as these soldiers would conduct a PCS move and attrit in the same fiscal year. Some further analysis is needed to determine if this is the case and if so to account for the additional flows by estimating the appropriate flow rates





from additional data and revising the transition matrix appropriately.



## VI. CONCLUSIONS AND RECOMMENDATIONS

A methodology has been developed to forecast future MACOM end strengths, thereby indicating unit readiness based on the personnel criteria stated in AR 220-1. By developing a transition matrix and repeating this model for each MOS, it may be possible to develop a methodology to forecast inventories and thus unit readiness for the entire Army.

The strength of such a model is in its value as a decision making tool for testing manpower policies at the Department of the Army level. For example, in the developmental stage of any major weapons system or vehicle, requirements for personnel support packages are developed. If it is known that force levels are to change in the near future, then such a model can be used to assist in a smooth transition of personnel. By establishing a personnel end strength goal as the future stock vector and using the current inventories as the current stock vector, the appropriate transition rates may be determined to assure attainment of the stated goal in an efficient manner.

In the previous example, required transition rates were derived analytically using vector matrix algebra. This model may also be used to study the effects of heuristically derived transition rates. A decision maker may want to know what the effect on force readiness might be if a policy were



implemented that would decrease promotion rates by ten percent throughout the Army. Insights to such questions are easily obtained as the analyst simply varies the appropriate parameter and runs the model the required number of years.

By running the model several years into the future, the decision maker is given insight into the steady state characteristics of the Army force strength. The model may be used to guide the decision maker in policy areas and requirements concerning promotion, recruitment, and PCS while maintaining a required force readiness level.

It is thought that historical data for three fiscal years is not sufficient to determine a trend in Army wide transition rates. Generally, data for the five most recent fiscal years can provide more stability while maintaining relevancy for future forecasting purposes.

In addition to the versatility of the model, as described above, this model is thought to be quite robust. Although the data appeared to be unpromising upon initial inspection, the results shown in Table VIII indicate that reasonably good forecasting results may still be obtained from it.

Finally, initial research in this methodology has revealed several areas that are in need of additional analysis. First, research needs to be conducted and additional data obtained to verify the value of the



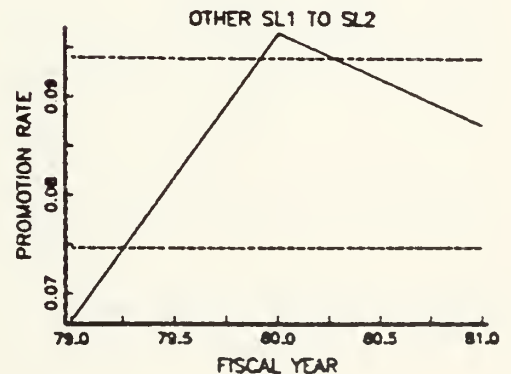
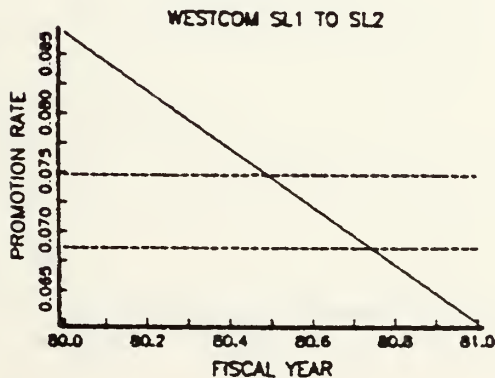
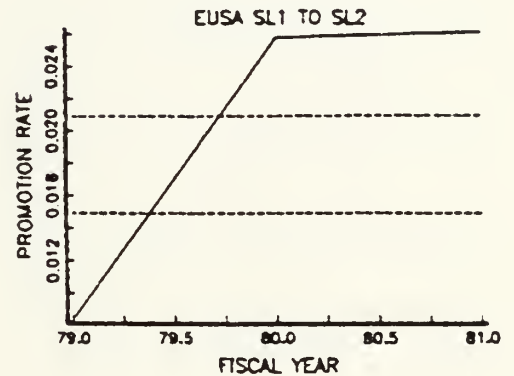
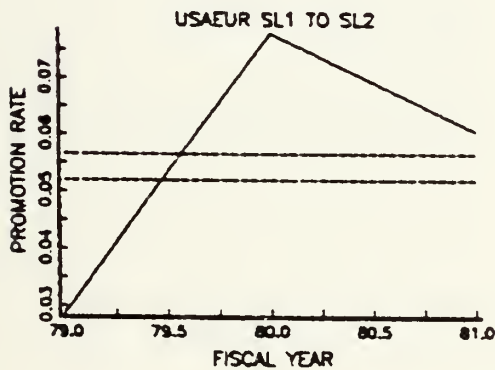
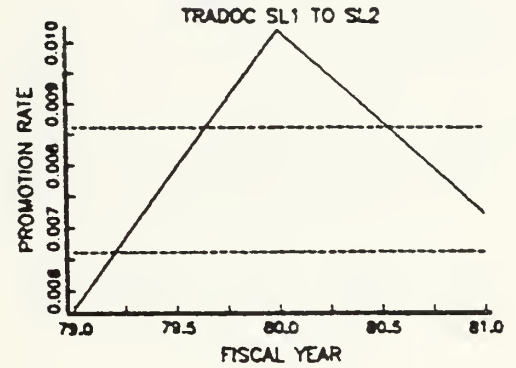
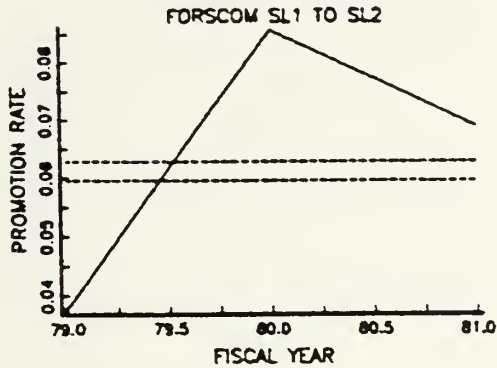
recruitment module. Additional areas of research related to this modeling effort are as follows:

1. expand this model to account for the more commonly known system of pay grades instead of the currently used skill levels;
2. develop a more efficient distribution algorithm for this model;
3. repeat this methodology for other MOS's and eventually all MOS's;
4. develop a methodology to forecast force readiness of commands subordinate to a MACOM, ultimately to UIC level;
5. develop this model into a software package; and to
6. develop a responsive interactive data base to support this model and similar manpower planning models.

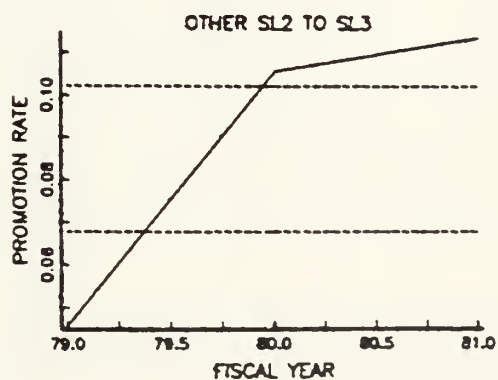
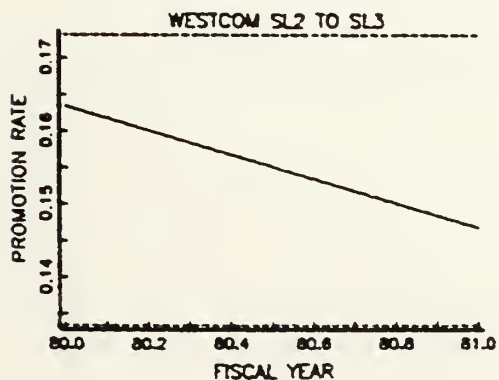
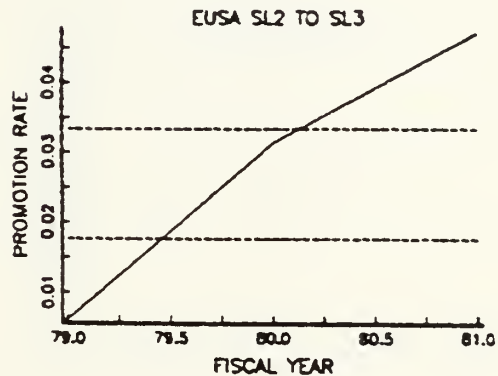
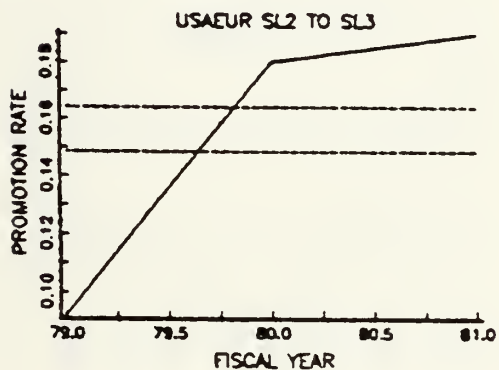
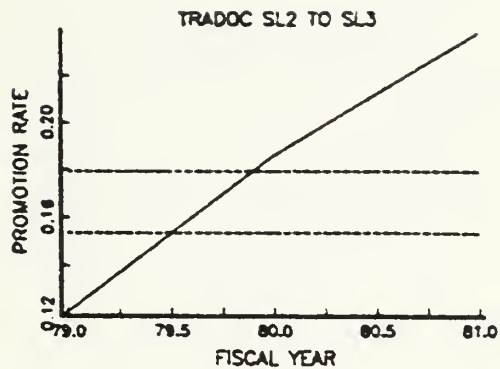
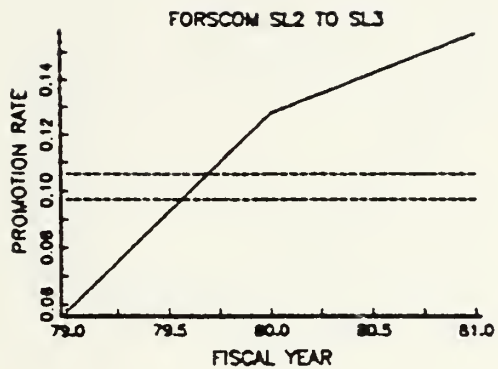




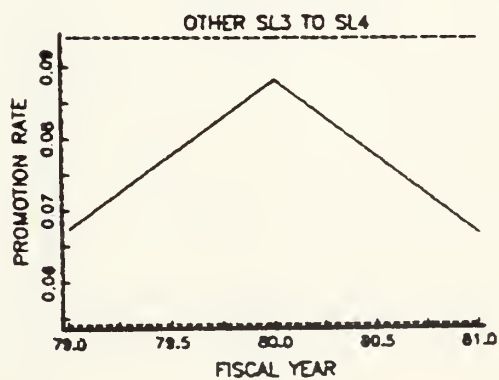
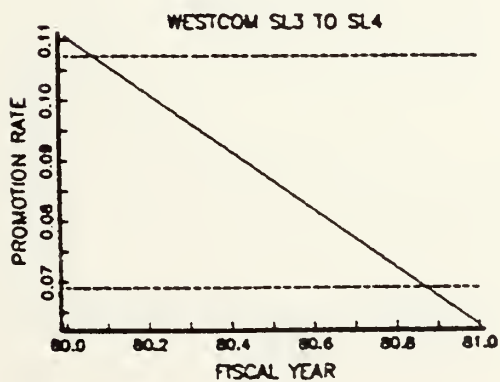
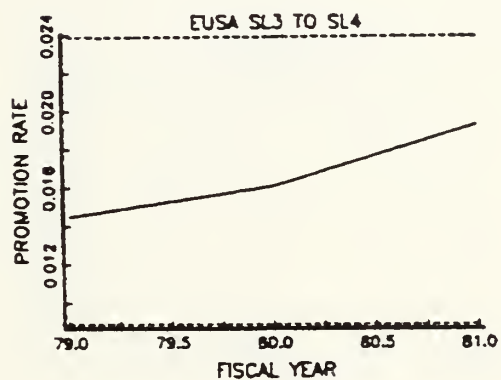
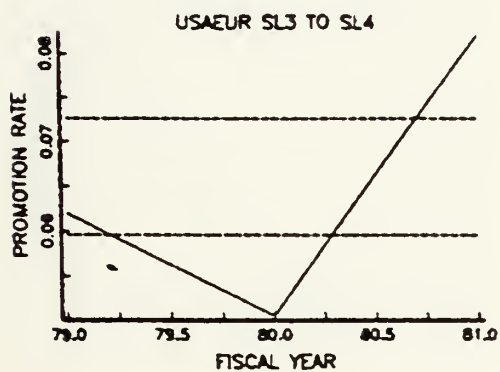
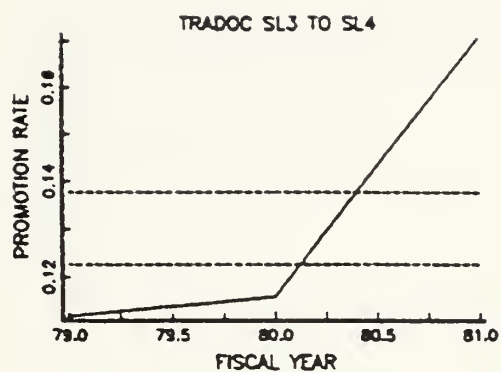
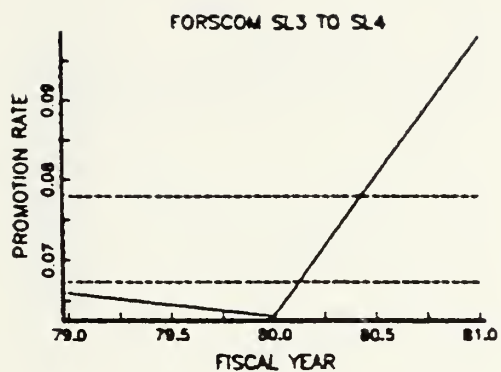
APPENDIX A: FY 1979 TO FY 1981 PROMOTION AND ATTRITION RATES  
FOR MOS 11B



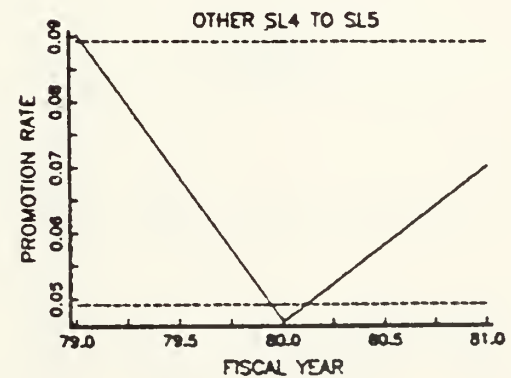
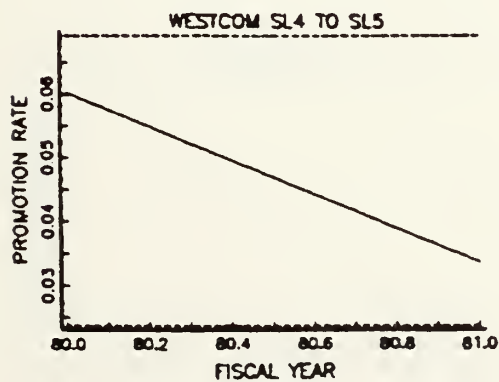
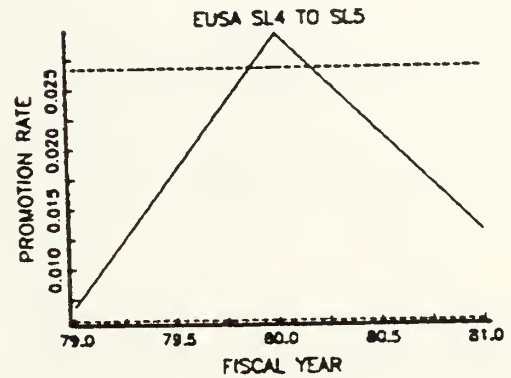
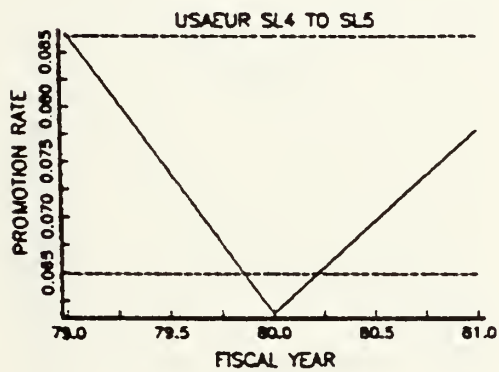
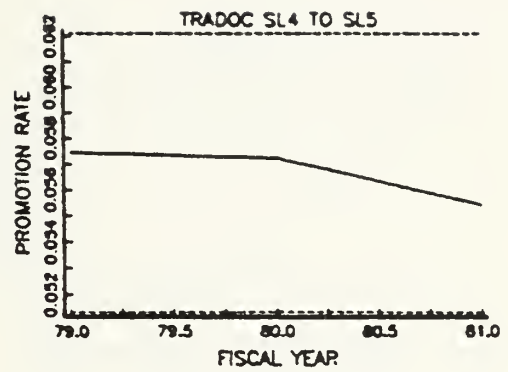
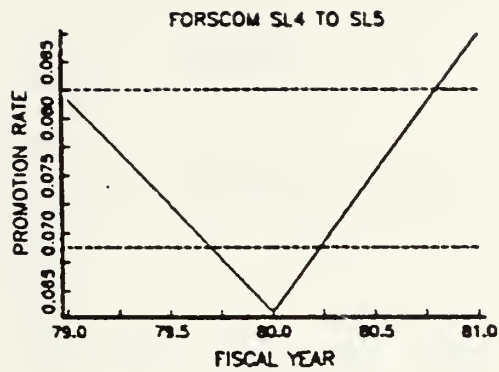






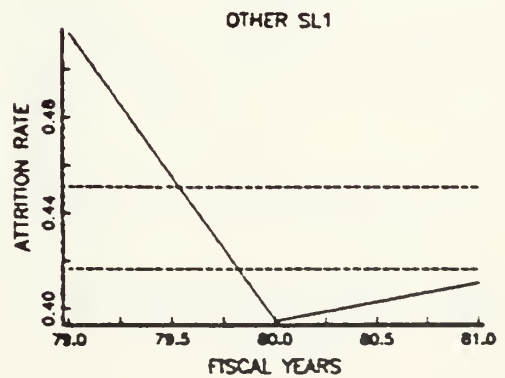
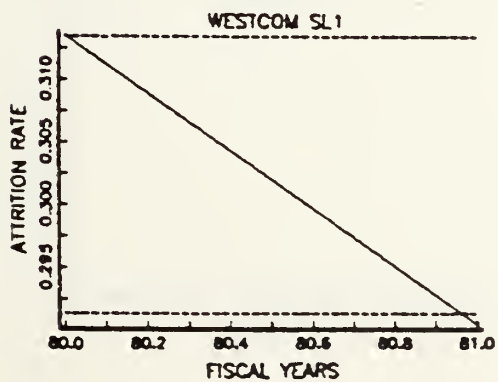
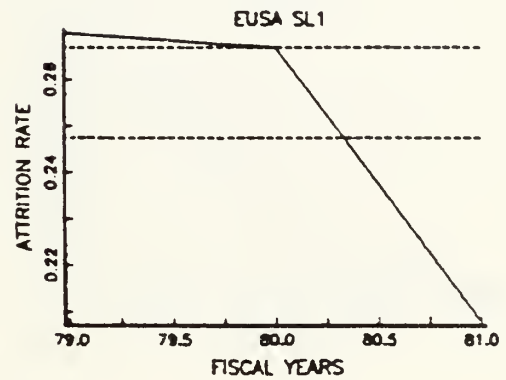
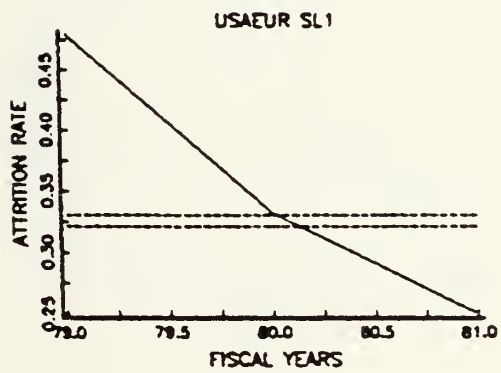
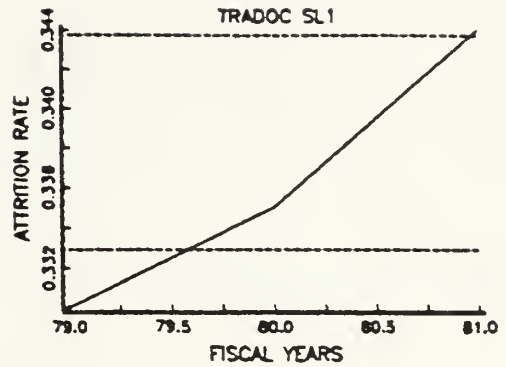
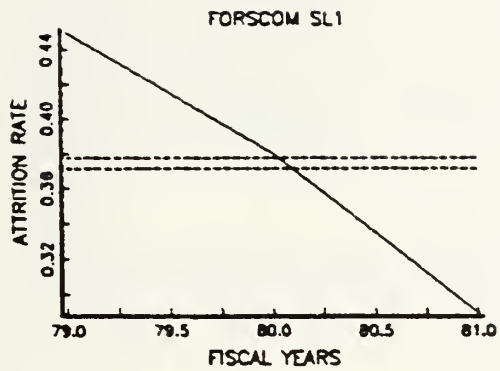




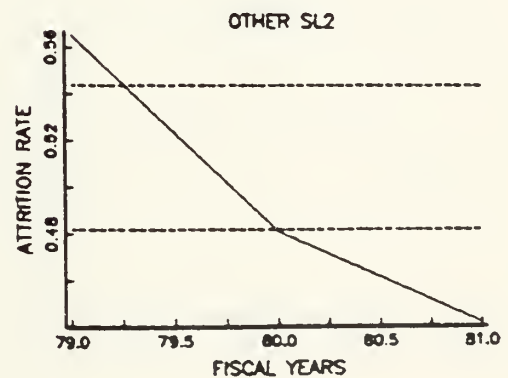
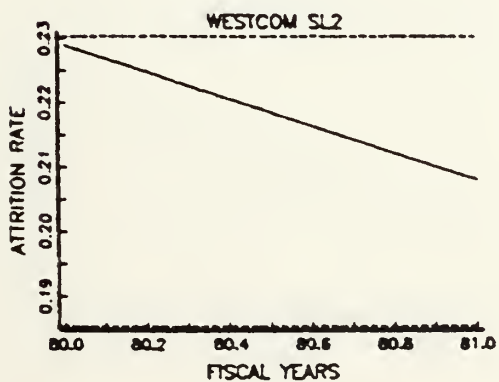
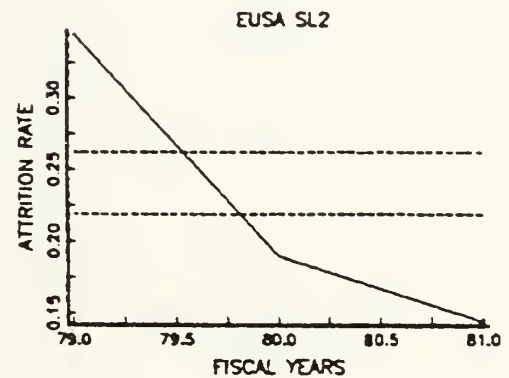
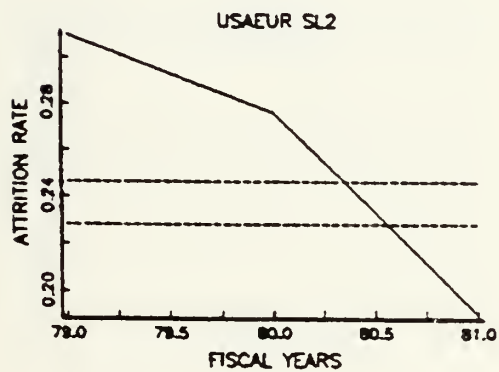
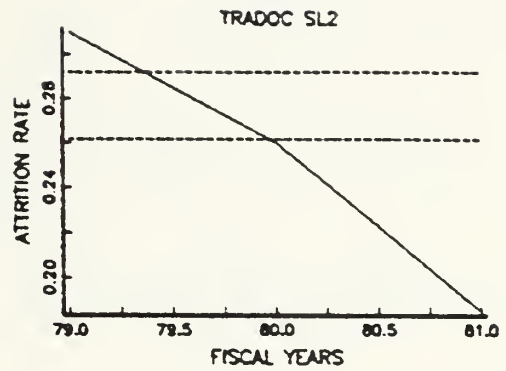
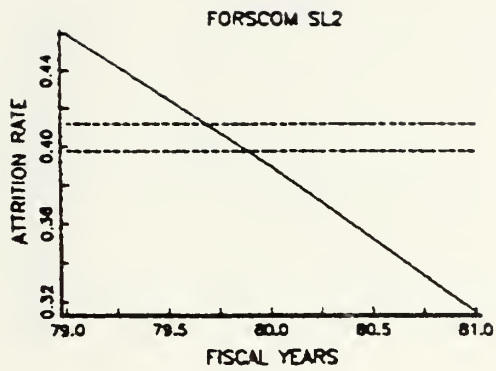




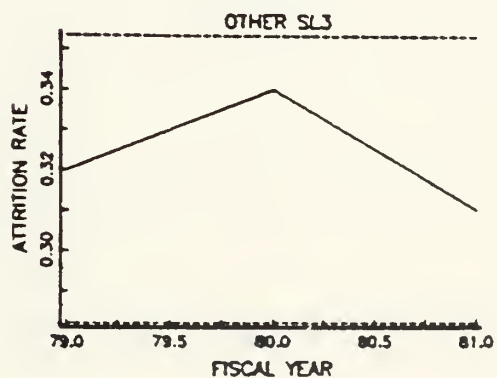
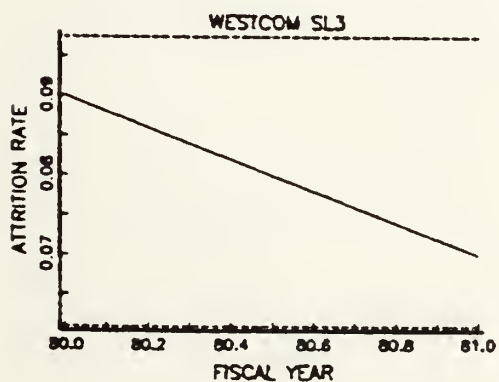
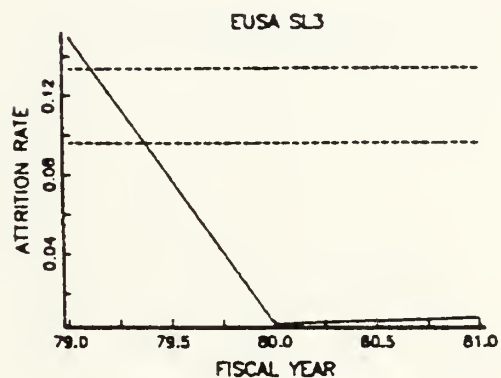
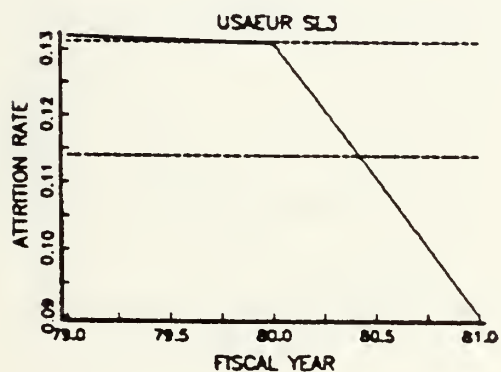
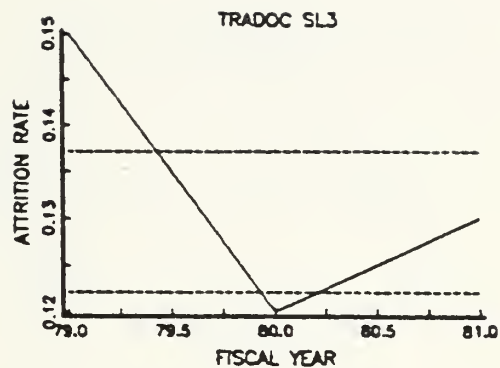
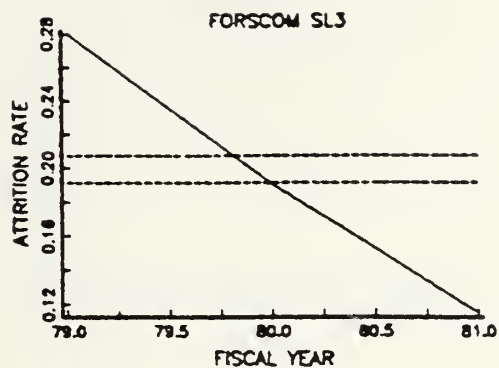




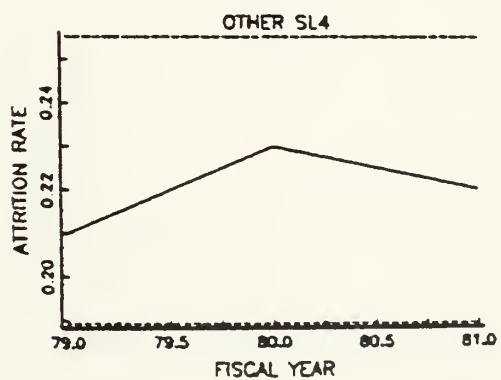
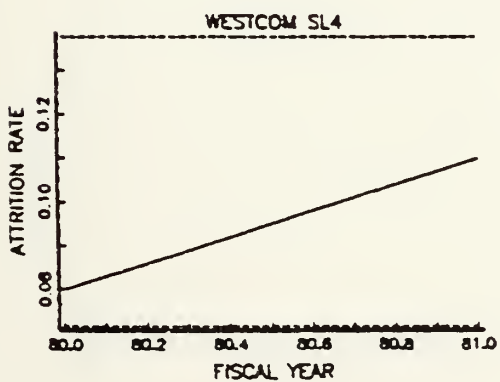
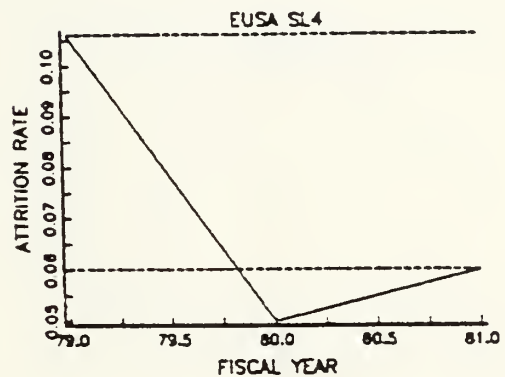
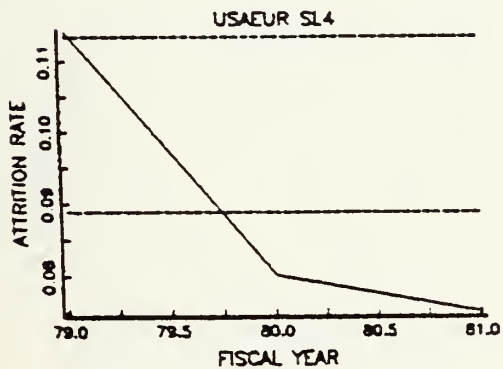
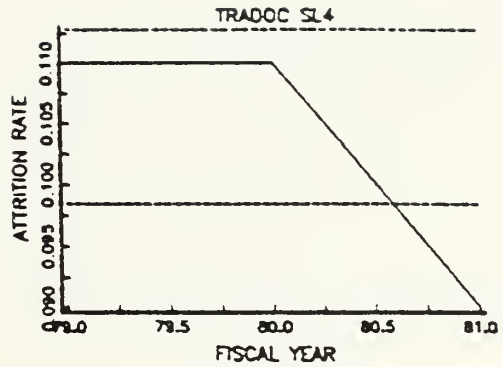
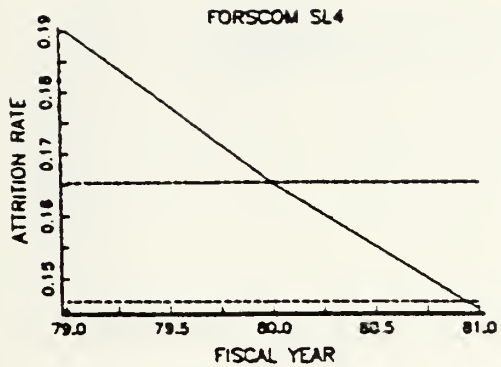






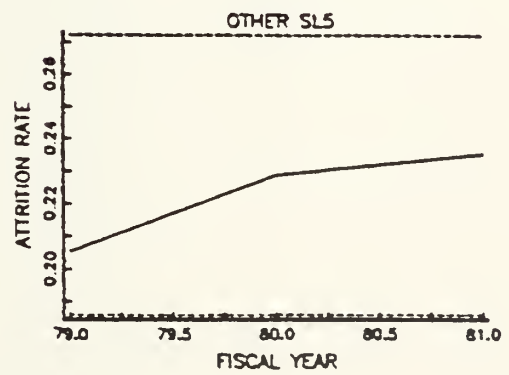
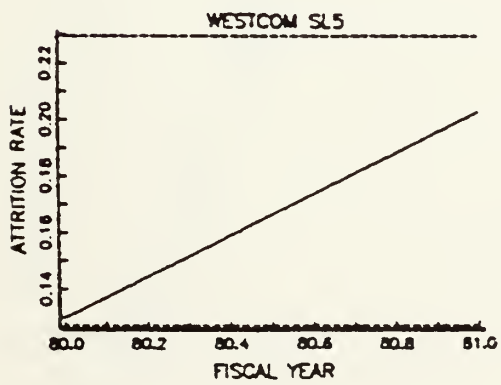
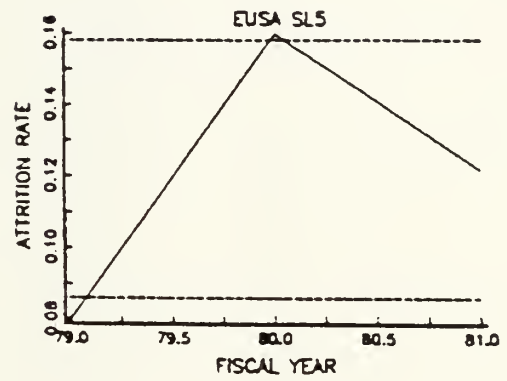
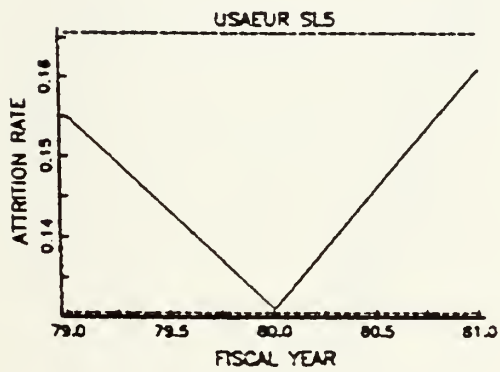
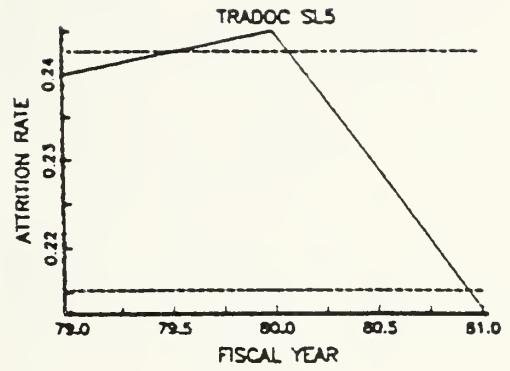
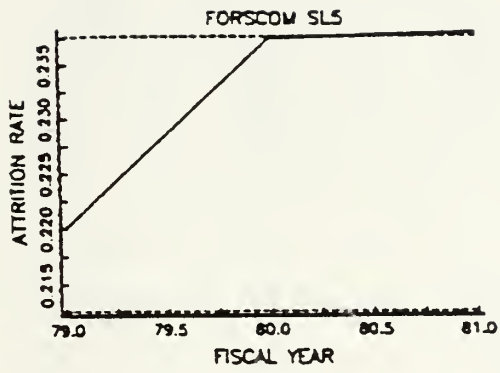














## APPENDIX B: TRANSITION MATRICES FOR MODELING METHODS ONE AND TWO

[illegible]



0.4785	0.3659	0	0	0.0114	0.013	0	0	0.0247	0.0945	0	0	0.0377	0.0039	0	0	0.0131	0.0124	0	0	0.0103	0.0145	0	0	0.006	0.0063	0	0	0
0	0.1274	0.1247	0	0	0	0.0176	0.009	0	0	0.0377	0.0141	0	0	0.0381	0.0031	0	0	0.0259	0.0026	0	0	0.0054	0.0037	0	0	0.0093	0.0071	0
0	0	0.0529	0.0151	0	0	0	0.0254	0.0166	0	0	0.0381	0.0031	0	0	0.0381	0.0031	0	0	0.0259	0.0026	0	0	0.0054	0.0037	0	0	0.0093	0.0071
0	0	0	0.0171	0.0045	0	0	0	0.0085	0.0053	0	0	0.0379	0.0176	0	0	0.0145	0.003	0	0	0.0094	0.003	0	0	0.0039	0	0	0.0114	0.002
0	0	0	0	0.014	0	0	0	0	0.0078	0	0	0.172	0	0	0.0123	0	0	0	0	0	0	0	0.0039	0	0	0	0	0.0075
0.1372	0.0029	0	0	0.0344	0.0021	0	0	0.1187	0.001	0	0	0	0.0149	0.001	0	0	0.0154	0	0	0	0.0154	0	0	0.013	0.0091	0	0	0
0	0.0534	0.0333	0	0	0.0372	0.0331	0	0	0.0071	0.0252	0	0	0.3174	0.0212	0	0	0.0088	0.0053	0	0	0.0088	0.0053	0	0	0.0104	0.0045	0	0
0	0	0.0154	0.0212	0	0	0.1772	0.1777	0	0	0.0431	0.0057	0	0	0.0404	0.0099	0	0	0.0177	0.0073	0	0	0.0177	0.0073	0	0	0.0133	0.0059	0
0	0	0	0.09	0.0034	0	0	0	0.0371	0.0058	0	0	0	0.0173	0.0017	0	0	0	0	0	0	0	0	0.0117	0.0034	0	0	0.0157	0.0022
0	0	0	0	0	0	0	0	0.0442	0	0	0	0	0.0159	0	0	0	0.0179	0	0	0	0	0	0.0125	0	0	0	0.0153	
0.1154	0.0145	0	0	0.0095	0	0	0	0	0.0063	0.0024	0	0	0.0019	0.0003	0	0	0.0056	0.0004	0	0	0.0056	0.0004	0	0	0.0082	0.0003	0	0
0	0.1125	0.0075	0	0	0.0111	0.0155	0	0	0.1628	0.1888	0	0	0.0004	0.0004	0	0	0.0004	0.0004	0	0	0.0004	0.0004	0	0	0.0127	0.0025	0	0
0	0	0.15	0.0132	0	0	0	0.0046	0.0194	0	0	0.0055	0.037	0	0	0.0012	0	0	0	0	0	0	0	0	0	0	0.014	0.0012	0
0	0	0	0.105	0.0011	0	0	0	0.1201	0.0194	0	0	0.0057	0.0073	0	0	0	0.0016	0	0	0	0	0	0	0	0	0	0.0114	0.0018
0.0539	0.123	0	0	0.1362	0	0	0	0.1103	0	0	0	0	0.0156	0	0	0	0.0076	0.0041	0	0	0.0004	0	0	0	0	0.0015	0.0024	0
0	0.275	0.2124	0	0	0.1015	0.0254	0	0	0	0	0	0	0	0	0	0	0.0179	0.0472	0	0	0	0	0	0	0	0	0	0
0	0	0.1353	0.011	0	0	0	0.1707	0.0095	0	0	0.0077	0	0	0	0	0	0.0779	0.0123	0	0	0	0	0.0037	0	0	0	0	0
0	0	0.1353	0.0521	0	0	0	0.1876	0.0197	0	0	0	0	0	0	0	0	0	0.0015	0.0111	0	0	0	0	0.0065	0	0	0	0
0	0	0	0	0.1754	0	0	0	0.1625	0	0	0	0	0	0	0	0	0	0	0.0097	0	0	0	0	0	0	0	0	0
0.0747	0.0147	0	0	0.0125	0.0017	0	0	0.0011	0	0	0	0	0.0005	0	0	0	0	0	0	0.0007	0.0011	0	0	0	0	0	0	0.023
0	0.107	0.0722	0	0	0.0474	0.0199	0	0	0.0035	0	0	0	0	0	0	0	0	0	0	0.0057	0.0011	0	0	0	0	0	0	0
0	0	0.1781	0.0135	0	0	0.11	0.0125	0	0	0.0049	0	0	0	0	0	0	0.0145	0.0097	0	0	0.0054	0.0011	0	0	0	0.0097	0	0
0	0	0.1149	0.0228	0	0	0	0.1495	0	0	0	0	0.0112	0	0	0	0	0	0	0	0.0073	0.0011	0	0	0	0	0	0.0037	0
0	0	0	0	0.1731	0	0	0	0.1116	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0327	0
0.0417	0.0049	0	0	0.0071	0.0041	0	0	0.0154	0.0027	0	0	0	0.0041	0.0019	0	0	0	0	0	0	0	0	0	0	0	0	0	0.012
0	0.0135	0.0119	0	0	0.0075	0.0157	0	0	0.001	0.0029	0	0	0	0.0025	0.0127	0	0	0	0	0.0073	0.0011	0	0	0	0	0	0.0037	0
0	0	0.0344	0.0127	0	0	0.0097	0.0045	0	0	0.0042	0.0041	0	0	0.0242	0.0041	0	0	0	0	0.0177	0.0041	0	0	0	0	0	0.0037	0.0067
0	0	0	0.1111	0.0059	0	0	0.0535	0.0039	0	0	0.0149	0.0053	0	0	0.0117	0.0053	0	0	0	0.0175	0	0	0	0	0	0	0.0037	0.0067
0	0	0	0	0.1144	0	0	0	0	0	0	0	0	0.0137	0	0	0	0	0	0.0047	0	0	0	0	0	0	0	0	0.0037
0	0	0	0	0.1144	0	0	0	0	0	0	0	0	0.0137	0	0	0	0	0	0.0047	0	0	0	0	0	0	0	0	0.0037



## LIST OF REFERENCES

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